Novel Analytical Framework for 6G Fluid Antenna Systems Utilizing Matrix Approximation

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***Abstract*—In 6G mobile communication systems, fluid antenna systems (FAS) have emerged as a promising technology for achieving additional diversity within limited spatial constraints. Although extensive research has been conducted on the performance analysis of FAS, existing analytical frameworks suffer from significant limitations, including inaccurate performance estimation and increased computational complexity as the number of antenna ports increases. This paper proposes a novel analytical framework utilizing matrix approximation to evaluate the outage probability of FAS, an emerging technology for 6G wireless communications. The framework effectively captures the intricate correlation structure of FAS channels while maintaining high analytical precision. Specifically, Jake's model-based covariance matrix, which characterizes the statistical properties of FAS channels, is approximated using the exponential correlation matrix. This approximation enables the derivation of closed-form expressions for the cumulative distribution function (CDF) of correlated FAS channels and the system's outage probability. The obtained results demonstrate excellent agreement with simulation results, even as the number of antenna ports increases. Notably, the proposed approach avoids computationally expensive multiple integrals, allowing efficient evaluation of outage probabilities through simple summation. This study offers rigorous analytical insights and provides significant contributions to the design and analysis of correlated communication systems.**

***Index Terms*—6G, fluid antenna systems (FAS), matrix approximation, outage probability.**

I. Introduction

Multiple-input multiple-output (MIMO) technology is a cornerstone of 6G wireless networks, enhancing capacity and reliability through multiple antennas at the transmitter and receiver [1]. Advanced approaches like cell-free massive MIMO and holographic MIMO optimize frequency efficiency and spatial resource utilization but face challenges due to high hardware costs associated with dedicated RF chains per antenna [2]. Recent innovations, such as liquid metal antennas using gallium-tin alloys, address these issues by reducing size, weight, and complexity [3].

Received 4 January 2025; revised 5 May 2025; accepted 29 May 2025. Date of publication 4 June 2025; date of current version 20 November 2025. This work was supported in part by the Institute for Information and Communications Technology Promotion (IITP) Grant funded by the Korea Government (MSIP, Development of Cube Satellites Based on Core Technologies in Low Earth Orbit Satellite Communications) under Grant RS-2024-00396992, in part by IITP Grant funded by the MSIT (Augmented Beam-Routing: Carom-MIMO) under Grant 2021-0-00486, and in part by the Research Program at the Korea Science Academy of KAIST funded by the Korean Government (Ministry of Science and ICT). The review of this article was coordinated by Dr. Wee Kiat New. *(Ye Rim Lee and Juyeong Baek contributed equally to this work.) (Corresponding author: Bang Chul Jung.)*

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Digital Object Identifier 10.1109/TVT.2025.3576370

Building on this trend, fluid antenna systems (FAS) have emerged as a groundbreaking solution. These antennas utilize software-controlled fluid to dynamically adjust their shape, size, and position [4]. For conventional MIMO systems, it is well-established that antennas must be spaced at least half a wavelength ($\lambda/2$) apart to achieve diversity gain [5]. However, recent findings [4] revealed that even a minimal spacing of $\lambda/10$ can result in a significant difference between deep fade and excellent reception. By leveraging the intrinsic properties of the wireless channel, FAS allows precise adjustments to the antenna's location, enhancing signal quality. This adaptability makes FAS a highly promising technology for next-generation wireless networks. Recent studies have highlighted the versatility of FAS in various next-generation wireless applications [6], [7], [8]. FAS enables dynamic reconfiguration of antenna properties, making it highly suitable for technologies such as integrated sensing and communications (ISAC) [7] and non-orthogonal multiple access (NOMA) [8].

Several studies have focused on the mathematical analysis of fluid antenna systems (FAS) under correlated fading channels, especially in terms of outage probability [4], [9], [10], [11], [12], [13], [14]. In [4], the authors mathematically analyzed the outage probability of FAS, but their assumption of spatial correlation existing only between the first antenna port and the others is uncommon in the literature. In [9], a refined correlation model that accounts for the correlation among all antenna ports in FAS was proposed, along with an approximation of the outage probability. However, the analysis presented in this work does not yield a *closed-form* solution and aligns well with simulation results only when the number of antenna ports is sufficiently large. In [10], the outage probability of FAS was approximated in a closed form by reducing the rank of the covariance matrix that represents the spatial correlation among antenna ports in FAS. However, the validity of the approximated outage probability was demonstrated through computer simulations only for cases with a small number of antenna ports, such as three or four. This limitation may be due to the increasing complexity of plotting the approximated equations as the number of antenna ports grows. In [11], [12], [13], copula theory was applied to approximate the correlation structure among the channels at different ports of FAS, providing an accurate estimation of the outage probability under specific system parameters. For instance, the approximations closely align with computer simulations when the size of the FAS is relatively small [12] or when a higher spatial correlation exists among the ports [13]. In [14], the outage probability of a reconfigurable intelligent surface–assisted FAS system was analyzed by approximating the correlation matrix as a block-diagonal matrix, although a closed-form expression was not derived.

In this paper, we derive a novel closed-form expression for the outage probability of FAS by approximating the spatial covariance matrix using an exponential correlation matrix.

II. System Model

We consider a system comprising a transmitter equipped with a single fixed antenna and a receiver equipped with a single fluid antenna.[1]

[1]As considered in [8], we also utilize this 2D FAS model. However, we acknowledge that this model is somewhat oversimplified from both a practical and forward-looking perspective. Nevertheless, this paper focuses on this system model. As previously mentioned, universal analysis within this model has not

The receiver is a linear antenna with a length of $W\lambda$, where W denotes the antenna's length parameter, and λ represents the wavelength. Inside the antenna is a movable fluid that can be positioned at N predefined, equally spaced locations, referred to as ports. Following the convention in the literature [9], [10], the signal vector at the receiver, $\mathbf{y} \in \mathbb{C}^{N\times 1}$, is given by

$$\mathbf{y} = \mathbf{h}x + \mathbf{n}, \tag{1}$$

where x denotes the transmitted signal and $\mathbf{n} \in \mathbb{C}^{N\times 1}$ denotes the additive white Gaussian noise (AWGN) and it is assumed that $\mathbf{n} \sim \mathcal{CN}(\mathbf{0}, N_0\mathbf{I}_N)$. Here, N_0 and $\mathbf{I}_N$ denote noise variance and $N \times N$ identity matrix, respectively. It is important to note that although there are multiple candidate locations for the antenna element, only a single fluid antenna port becomes active, as only one antenna element moves to one of the available locations to receive the signal.

The wireless channel from the transmitter to the receiver is defined as $\mathbf{h} \triangleq [h_1, h_2, \ldots, h_N]^T \in \mathbb{C}^{N\times 1}$, where $\mathbf{h} \sim \mathcal{CN}(\mathbf{0}, \mathbf{R})$ and $h_k \sim \mathcal{CN}(0, \sigma^2)$. Here, $\mathbf{R} \in \mathbb{R}^{N\times N}$ represents the spatial covariance matrix, such that $\mathbb{E}[\mathbf{h}\mathbf{h}^H] = \mathbf{R}$. Consistent with existing studies on FAS [4], [9], [10], [13], we assume that the spatial correlation between the k-th and the l-th ports is modeled using Jakes' model as follows:

$$\begin{aligned} R_{k,l} &\triangleq \mathsf{Cov}(h_k, h_l) = \sigma^2 J_0\left(2\pi \frac{\Delta d_{k,l}}{\lambda}\right) \\ &= \sigma^2 J_0\left(\frac{k-l}{N-1}W\right) \quad \text{for } k, l \in \{1, \ldots, N\}, \end{aligned} \tag{2}$$

where $\Delta d_{k,l}$ and $J_0(.)$ denote the distance between the k-th port and the l-th port and the zero-order Bessel function of the first kind, respectively.[2] Here, σ accounts for the effects of large-scale fading, which, without loss of generality, is assumed to be 1in this paper. Additionally, the FAS is assumed to activate the optimal single port that maximizes the signal envelope, ensuring optimal communication performance, as follows:

$$M = \arg\max_k \{|h_1|, |h_2|, \ldots, |h_N|\}. \tag{3}$$

Consequently, the resultant signal-to-noise ratio (SNR) at the FAS receiver is given by $\mathsf{SNR} = |h_M|^2\Theta$, where Θ represents the average transmit SNR, defined as $\Theta \triangleq P/N_0$. The outage event is defined as the condition where the received SNR falls below the threshold required for achieving a target data rate R, denoted by

$$\{\log_2(1 + \mathsf{SNR}) < \mathsf{R}\} = \left\{|h_M| < \sqrt{\gamma_{th}/\Theta}\right\}, \tag{4}$$

yet been conducted, and since this model represents the most basic form, we believe that solving the problem within this framework is of primary importance.

[2]By definition, $\mathbf{R}$ is a symmetric and positive semi-definite matrix. However, its positive semi-definiteness may not hold for arbitrary values of N and W. To ensure this property, we carefully select N and W such that they satisfy the positive semi-definite condition as required by the definition of the covariance matrix in this paper.

where $\gamma_{th} \triangleq 2^R - 1$. The outage probability $\mathsf{P}_{\mathsf{out}}(\gamma_{th})$ can be determined using the cumulative distribution function (CDF) of $|h_M|$, denoted as

$$\begin{aligned} \mathsf{P}_{\mathsf{out}}(\gamma_{th}) &= F_{|h_M|}\left(\sqrt{\gamma_{th}/\Theta}\right) \\ &\triangleq \Pr\left(|h_M| \le \sqrt{\gamma_{th}/\Theta}\right), \end{aligned} \tag{5}$$

where, $\Pr(\cdot)$ denotes the probability of an event.

III. Outage Probability Analysis

In this section, we conduct a mathematical analysis of the outage probability of the FAS. Let $|h_1|, |h_2|, \ldots, |h_N|$ be denoted by $\bar{h}_1, \bar{h}_2, \ldots, \bar{h}_N$, and define $\bar{\mathbf{h}} \triangleq [\bar{h}_1, \bar{h}_2, \ldots, \bar{h}_N]^T$. Determining the joint probability distribution of $(\bar{h}_1, \bar{h}_2, \ldots, \bar{h}_N)$ is typically very challenging. However, if the covariance matrix $\mathbf{R}$ has a tridiagonal inverse, the N-variate Rayleigh probability density function (PDF) for correlated channels can be expressed as [15]

$$\begin{aligned} f_{\bar{\mathbf{h}}}(r_1, r_2, \ldots, r_N) &= |\mathbf{W}| r_N e^{-w_{N,N} r_N^2/2} \\ &\times \prod_{k=1}^{N-1} \left[r_k e^{-w_{k,k} r_k^2/2} I_0\left(|w_{k,k+1}| r_k r_{k+1}\right)\right], \end{aligned} \tag{6}$$

where $\mathbf{W} = \mathbf{R}^{-1}$, $w_{i,j}$ denotes the element located in the i-th row and j-th column of $\mathbf{W}$ $(1 \le i, j \le N)$, and $I_0(.)$ denotes the modified zero-order Bessel function of the first kind, respectively. The N-variate Rayleigh joint CDF is expressed as

$$\begin{aligned} &F_{\bar{\mathbf{h}}}(R_1, R_2, \ldots, R_N) \triangleq \Pr\left(\bar{h}_1 \le R_1, \ldots, \bar{h}_N \le R_N\right) \\ &= \int_0^{R_1}\int_0^{R_2}\cdots\int_0^{R_N} f_{\bar{\mathbf{h}}}(r_1, r_2, \ldots, r_N)\, dr_1 dr_2 \cdots dr_n. \end{aligned} \tag{7}$$

Then, a closed-form expression for the correlated Rayleigh CDF, as presented in (8) at the bottom of the page, is derived [15]. When $R_1 = R_2 = \cdots = R_N = \sqrt{\gamma_{th}/\Theta}$, (5) and (7) become equivalent. The expression (9), provided at the bottom of the next page, includes an infinite series of the incomplete Gamma function defined as $\gamma(a, z) \triangleq \int_0^z e^{-t}t^{a-1}dt$. However, if the inverse of the covariance matrix $\mathbf{W}$ does not exhibit the tridiagonal property, (8) cannot be applied. To address this limitation, we propose a novel *matrix approximation* method to transform $\mathbf{R}$ into a form whose inverse possesses the tridiagonal property. Before introducing the proposed matrix approximation method, we first present the existing approach, known as Green's matrix approximation [15], as follows. This method computes the matrix $\mathbf{C}$, which closely approximates the elements of $\mathbf{R}$, with the condition that the inverse of the resulting Green's matrix must exhibit tridiagonal properties.

$$\begin{aligned} F_{\bar{\mathbf{h}}}(R_1, R_2, \ldots, R_N) = |\mathbf{W}| &\times \sum_{i_1=0}^{\infty}\sum_{i_2=0}^{\infty}\cdots\sum_{i_{N-1}=0}^{\infty} \frac{|w_{1,2}|^{2i_1}|w_{2,3}|^{2i_2}\cdots|w_{N-1,N}|^{2i_{N-1}}}{w_{1,1}^{i_1+1} w_{2,2}^{i_1+i_2+1}\cdots w_{N-1,N-1}^{i_{N-2}+i_{N-1}+1} w_{N,N}^{i_{N-1}+1}} \left(\prod_{j=1}^{N-1}\frac{1}{i_j!\Gamma(i_j+1)}\right) \\ &\times \gamma\left(i_1+1, \frac{1}{2}w_{1,1}R_1^2\right)\gamma\left(i_1+i_2+1, \frac{1}{2}w_{2,2}R_2^2\right)\cdots\gamma\left(i_{N-2}+i_{N-1}+1, \frac{1}{2}w_{N-1,N-1}R_{N-1}^2\right) \\ &\times \gamma\left(i_{N-1}+1, \frac{1}{2}w_{N,N}R_N^2\right) \end{aligned} \tag{8}$$

Lemma 1: A symmetric, irreducible nonsingular matrix $\mathbf{A}$ is tridiagonal if and only if $\mathbf{A}^{-1} = \mathbf{C} = [C_{i,j}]$ is given by

$$\mathbf{C} = \begin{pmatrix} u_1 v_1 & u_1 v_2 & \cdots & u_1 v_N \\ u_1 v_2 & u_2 v_2 & \cdots & u_2 v_N \\ \vdots & \vdots & \ddots & \vdots \\ u_N v_2 & u_N v_2 & \cdots & u_N v_N \end{pmatrix}, \quad (10)$$

with u_i and $v_i, i = 1, \cdots N$, be two sequences of real numbers and $R_{i,j} = 1$ for $i = j$, we have $u_i v_i = 1$, for $1 \le i \le n$. Therefore, to ensure that the inverse of $\mathbf{C}$ has the tridiagonal property, each elements have to satisfy the condition as below,

$$R_{i,j} = u_{\min(i,j)} v_{\max(i,j)} = \frac{v_{\max(i,j)}}{v_{\min(i,j)}}. \quad (11)$$

If $\mathbf{R}^{-1}$ is tridiagonal, we must have $v_{\min(i,j)} R_{i,j} = v_{\max(i,j)}$. Equating $\mathbf{R}$ with $\mathbf{C}$, a nonlinear system equation is produced, as shown below,

$$\begin{array}{llll} v_2 = R_{2,1} v_1 & & & \\ v_3 = R_{2,1} v_1 & v_3 = R_{3,2} v_2 & & \\ & v_4 = R_{4,2} v_2 & v_4 = R_{4,3} v_3 & \\ \vdots & \vdots & v_5 = R_{5,3} v_3 & \ddots \ddots \\ v_n = R_{n,1} v_1 & v_n = R_{n,2} v_2 & \cdots\cdots & v_n = R_{n,n-1} v_{n-1} \end{array} \quad (12)$$

Levenberg-Marquardt, quasi-Newton, or conjugate gradient methods can be used to solve (12) for all $1 \le i < j \le N$. Since $u_i = 1/v_i$, $\mathbf{C}$ can be defined accordingly. Consequently, utilizing matrix $\mathbf{C}$ as the approximated covariance matrix facilitates closed-form mathematical analysis.

A. Geometric Matrix Approximation Method

In this section, we introduce a novel approach to approximate the covariance matrix using an exponential correlation model, which is extensively employed in MIMO systems to represent antenna correlations. In this model, correlation coefficients decrease exponentially with increasing distance between antennas. In applications such as uniform linear array antenna systems, it is reasonable to assume that the correlation coefficient matrix exhibits a Toeplitz structure, as correlations depend primarily on inter-antenna distances rather than their absolute positions. The following lemma asserts that, under a mild condition, a symmetric Toeplitz covariance matrix with a tridiagonal inverse must be an exponential correlation matrix.

Lemma 2: If $\mathbf{R}$ is an irreducible, symmetric, and nonsingular toeplitz matrix, then

$$R = [R_{i,j}] \text{ where } R_{i,j} = \rho^{|i-j|}, \text{ for some } \rho \ne \pm 1. \quad (13)$$

Proof: From $\Sigma_{i,i+1} = R_{i+1,i} = \frac{v_{i+1}}{v_i}$, for $1 \le i \le N-1$, we have $v_{i+1} = R_{i,i+1} v_i$. But, since $R_{i,i+1}$ is a constant (toeplitz), we have $v_{i+1} = \rho v_i$, where $\rho = R_{i,i+1} = R_{i+1,i}$ $(1 \le i \le N-1)$ with $\rho \ne 0$ (irreducible). From invertibility, $\rho \ne \pm 1$, $v_{i+1} = \rho^{i-1} v_i$, and $R_{i,j} = \frac{v_{\max(i,j)}}{v_{\min(i,j)}} = \rho^{|i-j|}$. Furthermore, $\mathbf{R} = [\rho^{|i-j|}]_{i,j=1}^N$ has a tridiagonal inverse, too. □

Let $\mathbf{C}$ denote the approximate exponential correlation matrix. It is reasonable to approximate the covariance matrix using this geometric model, $\rho^{|i-j|} = R_{i,j}$, to take ρ which minimizes

$$\sum_{i,j=1}^{N} \left(\rho^{|i-j|} - R_{i,j}\right)^2 = \|\mathbf{R} - \mathbf{C}\|_F^2, \text{ where } \mathbf{C} = \left[\rho^{|i-j|}\right]_{i,j=1}^{N}, \quad (14)$$

where $\|.\|$ denotes the frobenius norm.

A common form of the geometric covariance matrix is given by

$$\mathbf{C} = \begin{pmatrix} 1 & \rho & \cdots & \rho^{n-1} \\ \rho & 1 & \cdots & \rho^{n-2} \\ \vdots & \vdots & \ddots & \vdots \\ \rho^{n-1} & \rho^{n-2} & \cdots & 1 \end{pmatrix}, \quad (15)$$

where $0 \le \rho \le 1$ is the correlation coefficient. Since the inverse of the exponential correlation matrix exhibits a tridiagonal structure, we can utilize this covariance matrix in (8) to analyze the system's outage probability. It is assumed that two matrices $\mathbf{R}$ and $\mathbf{C}$ must have the same dimensions, i.e., $N \times N$ to find the value of ρ that minimizes the matrix mean squared error (MSE). The matrix mean squared error (MSE) is defined by (14), which can be input into software like MATLAB or other computational tools determine the optimal ρ. However, deriving a simple expression for ρ is challenging. As an alternative, we can minimize the MSE in the logarithmic domain as follows:

$$\begin{aligned} &\sum_{i,j=1}^{N} \left(\log|R_{i,j}| - \log \rho^{|i-j|}\right)^2 \\ &= \sum_{i,j=1}^{N} \left(\log|R_{i,j}|\right)^2 - 4\left(\sum_{k=1}^{N-1} k(N-k)\log|R_{1,1+k}|\right)\log\rho \\ &+ \frac{1}{6} N^2\left(N^2-1\right)(\log\rho)^2. \end{aligned} \quad (16)$$

After performing mathematical operations, we derive the following condition for ρ that minimizes the MSE in the logarithmic domain:

$$\log\rho = \frac{12}{N^2\left(N^2-1\right)} \sum_{k=1}^{N-1} k^{(N-k)} \log|R_{1,1+k}|. \quad (17)$$

With the optimal ρ obtained from (17), in this paper, we utilize the resulting exponential correlation matrix to determine the outage probability using (9).

$$\begin{aligned} F_{\bar{\mathbf{h}}}(R_1, \ldots, R_N) = &\left(1-\rho^2\right) \times \sum_{i_1, i_2, \ldots, i_{N-1}=0}^{\infty} \frac{g_2 \rho^{2(i_1+\cdots+i_{N-1})}}{\prod_{j=1}^{N-1}(i_j!)^2} \gamma\left(i_1+1, \frac{R_1^2}{(1-\rho^2)}\right) \\ &\times \left[\prod_{j=1}^{N-1} \gamma\left(i_{j-1}+i_j+1, \frac{R_j^2}{(1-\rho^2)}\left(1+\rho^2\right)\right)\right] \gamma\left(i_{N-1}+1, \frac{R_N^2}{(1-\rho^2)}\right) = \left(1-\rho^2\right)\sum_{n=0}^{\infty} b_n, \\ \text{where } b_n \triangleq &\sum_{i_1+\cdots+i_{N-1}=n}^{\infty} \frac{g_2 \rho^{2n}}{\prod_{j=1}^{N-1}(i_j!)^2} \left(\prod_{j=1}^{N-1} \gamma\left(i_{j-1}+i_j+1, \frac{(1+\rho^2)}{(1-\rho^2)} R_j^2\right)\right) \gamma\left(i_{N-1}+1, \frac{R_N^2}{1-\rho^2}\right) \end{aligned} \quad (9)$$

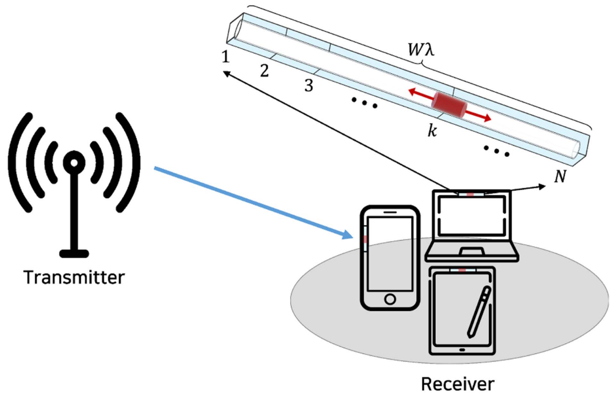

Fig. 1. System model of FAS.

B. Asymptotic Behavior of Outage Probability

This section investigates the outage probability behavior in the high-SNR regime. Assuming a high transmit SNR, we apply a Taylor series expansion to (7) under the condition $R_1 = R_2 = \cdots = R_N = \sqrt{\frac{\gamma_{th}}{\Theta}}$. Then, by considering only the first term of the Taylor series, we derive the asymptotic expression [10].

$$\mathsf{P}_{\text{out}}(\gamma_{th}) \approx \frac{1}{\det(\mathbf{R})} (\gamma_{th}/\Theta)^N \tag{18}$$

C. Computational Complexity of Mathematical Analysis

In (9), the presence of an infinite summation implies that as the number of antenna ports increases, the computational resources required–such as processing power and memory–escalate significantly. This escalation can render computations impractical within reasonable time frames or with available resources. To mitigate this issue, it is essential to reduce computational complexity by limiting the number of summation terms. Assuming $i_1 + i_2 + \cdots + i_{N-1} = n$ sets a specific threshold for the summation in (9), indicating that we perform the summation from $n = 1$ and up to n. To validate the proposed technique, we demonstrate that upon reaching a certain threshold, additional terms become negligible, yielding results equivalent to those obtained from an infinite summation, as formalized in the following theorem.

Theorem 1: In (9), we have

$$(1-\rho^2) b_n < \left(\frac{Ne}{2n^2}\right)^n \frac{2}{(4\pi)^{\frac{N}{2}}\sqrt{N-1}} \frac{1-\rho^2}{(\rho^2+1)^{N-2}}$$

for sufficiently large n so that $b_n = O(n^{(-2+\epsilon)n})$ for arbitrary constant $\epsilon > 0$, and the truncation error when summing up to n terms is at most

$$\left(\frac{Ne}{2n^2}\right)^n \frac{2}{(4\pi)^{\frac{N}{2}}\sqrt{N-1}} \frac{1-\rho^2}{(\rho^2+1)^{N-2}} \frac{Ne^{-1}}{(n+1)^2 - Ne^{-1}}.$$

Proof: Refer to Appendix. □

IV. Numerical Results

In this section, we validate our proposed analytical framework for FAS through computer simulations, assuming $R = 1\,(\text{bps/Hz})$ and $\sigma = 1$ for all figures. All figures presented below were generated using MATLAB simulations. Fig. 2 compares the outage probability performance of the proposed method, previous analytical approaches, and simulation results across varying transmit SNR conditions. The proposed method exhibits the closest alignment with the simulation results, demonstrating superior accuracy. The methods by Alouini (Approximated) [9] and Kai-kit (New Analysis) [10] also show relatively good agreement with the simulations.[3] In contrast, the Gaussian copula method [8] shows significant deviations, particularly at high SNR. Fig. 3 presents the outage probability performance of the proposed analytical framework compared to existing matrix approximation methods, including Green's matrix approximation and Alouini's approximation, alongside simulation results. The proposed analytical framework outperforms conventional matrix approximation methods across all values of N and W. While the Kai-kit method demonstrates a reasonable fit for $N = 3$ and $N = 4$, it can not be computed for $N \geq 5$ due to the increased complexity of its formula for larger N. Green's matrix approximation method [15] shows significant deviations from the simulation results, particularly at high SNR, due to matrix approximation errors. The Alouini approach performs reasonably well for $N = 4$; however, its accuracy declines for larger values of N, as observed in the cases of $N = 5$ and $N = 6$. Fig. 4 illustrates the outage probability of the proposed analytical framework for large N and W under varying transmit SNR. The proposed method consistently shows superior accuracy, closely matching the simulations across all

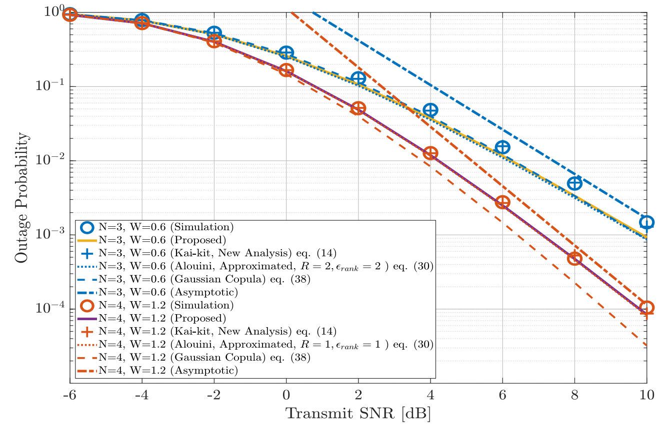

Fig. 2. Outage probability comparison between the proposed analytical framework and the conventional schemes [9], [10], [13].

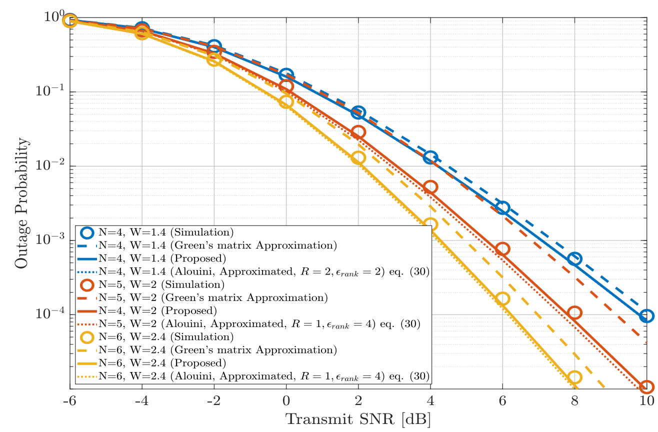

Fig. 3. Outage probability comparison between the proposed analytical framework and the conventional matrix approximation methods [9], [15].

[3]The method proposed in [9], [10], [13], exhibits slightly better performance than the proposed analysis in certain ranges of N and W, although such regimes are relatively rare.

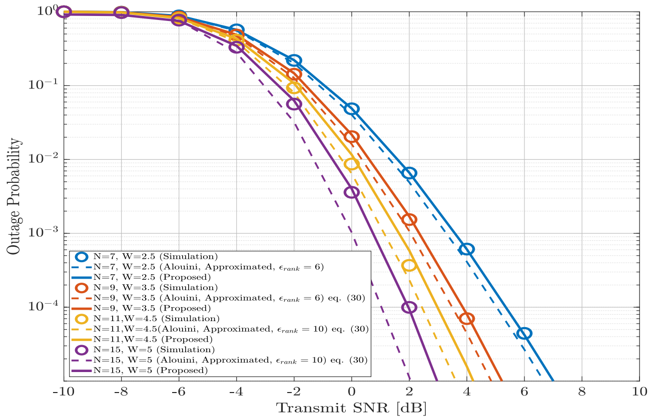

Fig. 4. Outage probability of the proposed analytical framework for relatively large N and W under varying transmit SNR conditions.

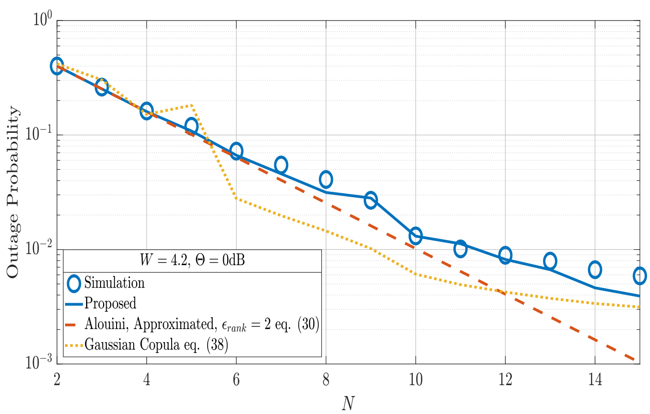

Fig. 5. Outage probability of the proposed analytical framework with respect to N.

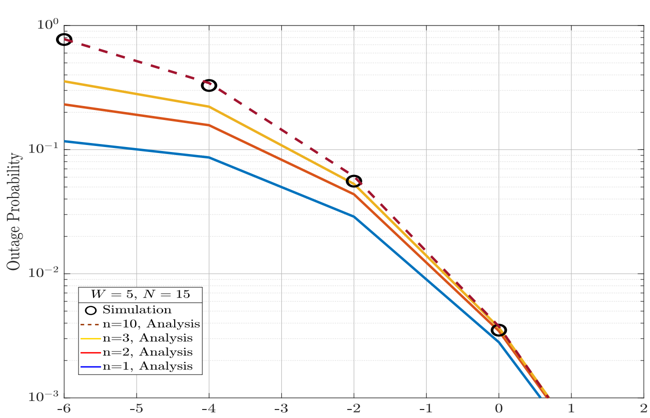

Fig. 6. Outage probability versus the number of summation terms.

configurations. In contrast, the Alouini approach exhibits increasing deviations as N and W grow. Fig. 5 illustrates the outage probability of the proposed analytical framework with respect to N, demonstrating that it accurately predicts the outage probability of the FAS system. This is shown in comparison with conventional analytical methods for various numbers of antenna ports. It can also be observed that the outage performance gradually saturates as N increases. Fig. 6 shows the outage probability versus transmit SNR for the case of $W = 5$ and $N = 15$, where the number of summation terms varies. In the low SNR region, the proposed method closely approximates the simulation results even with as few as $n = 10$ summation terms. In the high SNR region, accurate approximation is achievable with even fewer summation terms.

V. Conclusion

In this paper, we proposed a novel analytical framework for FAS, approximating the spatial covariance matrix as an exponential correlation matrix to derive a closed-form expression for the outage probability. Computer simulations demonstrate that the proposed framework significantly outperforms conventional methods across various antenna port counts and antenna lengths. Additionally, we reduced the computational complexity by introducing a threshold to limit the number of summations required for computing the outage probability in closed form. As part of future work, we plan to extend the proposed analytical framework to encompass multi-dimensional antenna configurations and multi-user scenarios. Such extensions will enable a more comprehensive and realistic performance evaluation in practical wireless communication environments.

Appendix
Proof of Truncation Error

Since incomplete gamma functions take values between 0 and 1, we need to estimate the magnitude of $g_2\rho^{2n} / \prod_{j=1}^{N-1}(i_j!)^2$ when n becomes large.

$$g_2\rho^{2n} = \frac{\rho^{2n}}{(\rho^2+1)^{i_1+2i_2+\cdots+i_{N-1}+(N-2)}} \leq \frac{\rho^{2n}}{(\rho^2+1)^{n+N-2}}, \tag{19}$$

$$\sum_{i_1+\cdots+i_{N-1}=n} \frac{1}{\prod_{j=1}^{N-1}(i_j!)^2} = \frac{1}{(n!)^2} \sum_{i_1+\cdots+i_{N-1}=n} \frac{(n!)^2}{\prod_{j=1}^{N-1}(i_j!)^2}$$
$$= \frac{1}{(n!)^2} \sum_{i_1+\cdots+i_{N-1}=n} \binom{N}{i_1\ i_2\ \cdots\ i_{N-1}}^2. \tag{20}$$

As derived in [16], the approximation of the summation is given by

$$\sum_{i_1+\cdots+i_{N-1}=n} \binom{N}{i_1\ \cdots\ i_{N-1}}^2 \approx (N-1)^{2n+\frac{N-1}{2}} (4\pi N)^{\frac{2-N}{2}} \tag{21}$$

and Stirling's formula states that $(n!)^2 \approx 2\pi n \cdot e^{2n\log n - 2n}$ as $n \to \infty$. Thus, (21) is derived as follows:

$$\frac{1}{(n!)^2} \sum_{i_1+\cdots+i_{N-1}=n} \binom{N}{i_1\ i_2\ \cdots\ i_{N-1}}^2 \tag{22}$$
$$\approx \frac{(N-1)^{2n+\frac{N-1}{2}} (4\pi N)^{1-\frac{N}{2}}}{e^{2n\log n - 2n} 2\pi N}$$
$$= \frac{(N-1)^{2n+\frac{N-1}{2}} e^{2n}}{n^{2n}} 2^{1-\frac{N}{2}} (2\pi N)^{-\frac{N}{2}}$$
$$= \left(\frac{(N-1)\,e^2}{n^2}\right)^n \frac{2}{\sqrt{N-1}} \left(\frac{N-1}{4\pi N}\right)^{\frac{N}{2}}. \tag{23}$$

Since the incomplete gamma functions in (9) is always less than 1, $(1-\rho^2)\frac{g_2\rho^{2n}}{\prod_{j=1}^{N-1}(i_j!)^2}$ becomes an upper bound of b_n. As $n \to \infty$, we obtain an approximate upper bound of (9) as follows:

$$\begin{aligned}
&\left(1-\rho^2\right)\frac{g_2\rho^{2n}}{\prod_{j=1}^{N-1}\left(i_j!\right)^2}\\
&\lesssim\left(\frac{\left(N-1\right)e^2}{n^2}\right)^n\left(\frac{N-1}{4\pi N}\right)^{\frac{N}{2}}\frac{2}{\sqrt{N-1}}\left(\frac{\rho^2}{\rho^2+1}\right)^n\frac{1-\rho^2}{\left(\rho^2+1\right)^{N-2}},\\
&<\left(\frac{\left(N-1\right)e^2}{n^2}\right)^n\frac{1}{4\pi}^{\frac{N}{2}}\frac{2}{\sqrt{N-1}}\left(\frac{1}{2}\right)^n\frac{1-\rho^2}{\left(\rho^2+1\right)^{N-2}}\\
&<\left(\frac{Ne}{2n^2}\right)^n\frac{2}{\left(4\pi\right)^{\frac{N}{2}}\sqrt{N-1}}\frac{1-\rho^2}{\left(\rho^2+1\right)^{N-2}}\triangleq c_n.
\end{aligned}\tag{24}$$

Then, we obtain the following asympotic behavior of c_n as $n\to\infty$.

$$\begin{aligned}
\frac{c_{n+1}}{c_n}&=\left(\frac{n^2}{\left(n+1\right)^2}\right)^n\frac{Ne}{\left(n+1\right)^2}=\left(1-\frac{1}{n+1}\right)^{2n}\frac{Ne}{\left(n+1\right)^2}\\
&\to e^{-2}\frac{Ne}{\left(n+1\right)^2}=\frac{Ne^{-1}}{\left(n+1\right)^2}
\end{aligned}\tag{25}$$

Finally, the truncation error in the proposed mathematical analysis of the outage probability for the FAS system, resulting from summing terms up to n, is upper bounded by

$$\begin{aligned}
\sum_{k=n+1}^{\infty}(1-\rho^2)b_k&<\sum_{k=n+1}^{\infty}c_k=\sum_{k=n+1}^{\infty}c_n\frac{c_{n+1}}{c_n}\cdots\frac{c_k}{c_{k-1}}\\
&\lesssim\sum_{k=n+1}^{\infty}c_n\left(\frac{Ne^{-1}}{(n+1)^2}\right)^{k-n}\\
&=c_n\frac{\left(\frac{Ne^{-1}}{(n+1)^2}\right)}{1-\frac{Ne^{-1}}{(n+1)^2}}=c_n\frac{Ne^{-1}}{(n+1)^2-Ne^{-1}}.
\end{aligned}$$

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