

Random Access Games With Cost of Waiting for Uplink NOMA Systems

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Abstract—In power domain non-orthogonal multiple access (NOMA) used for uplink random access (RA) systems, users need to control transmit power such that the received power at the base station can be one of the predetermined values. Thus, selecting one of the predetermined values incurs different transmission costs. Furthermore, users can often withdraw their (re)transmission, which involves a waiting (or regret) cost. This letter analyzes N -user non-cooperative game of uplink NOMA RA systems, where waiting cost is taken into account. As results, we show the condition, under which a unique mixed-strategy Nash equilibrium (MNE) exists and its efficiency with respect to social welfare maximization (SWM).

Index Terms—Non-orthogonal multiple access, random access, non-cooperative game, successive interference cancellation (SIC), waiting cost.

I. INTRODUCTION

RECENTLY, power-domain non-orthogonal multiple access (NOMA) [1] techniques have been applied for uplink random access (RA) protocol [2]–[6]. In the NOMA RA protocol, each user adjusts the transmit power such that the packet is received with a predetermined power level at a base station (BS) when he sends the packet to the BS over a shared wireless channel. When the BS receives the packets from users, it tries to decode them with a well-known successive interference cancellation (SIC) technique from the packet with the highest received power level to that with the lowest received power. If all packets from the users are received with a unique power level at the BS, then all the packets are successfully decoded, which enhances throughput (packets/slot) of the RA system.

The previous studies investigated uplink NOMA RA system with multiple target power levels in terms of throughput, average access delay, and stability [2]–[4]. In addition, game theoretic approaches were considered in [5], [6], where users selfishly maximize their payoff in attempting NOMA RA. In particular, an achievable payoff region with *two-user* non-cooperative game was characterized in [5]; that is, how much payoff two users can realize. In [6], N -user game was

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examined particularly when the NOMA RA system employs multiple (orthogonal) channels.

Compared to [5], [6], we consider *the cost of waiting* in N -user NOMA RA games such that the users will get a certain penalty if they do not (re)transmit. This corresponds to the scenario that the users deliver the information under a delay constraint for Internet-of-Things (IoT) with real-time control, e.g., vehicular networks. Thus, it is important to find how selfishly users interact to maximize their payoff in the presence of the cost of waiting with respect to a mixed-strategy Nash equilibrium (MNE). Key contribution of this letter is to characterize NOMA RA with non-cooperative game and compare it with social welfare maximization (SWM).

II. SYSTEM MODEL

A. NOMA RA System

In the uplink NOMA RA system, N contending users share the wireless channel to send their packet to the BS located at the center of service area. Time is divided into slots of a constant size and the time duration of the packet is assumed to be equal to that of each slot. We assume time division duplex (TDD) as in 5G new radio (NR) systems, which enable to facilitate uplink channel estimation at users by observing the downlink reference signals from the BS. Hence, each user is assumed to obtain its channel gain before transmitting a packet in the uplink.

Let us denote by Y and P_i the channel gain of a user and the target (received) power at the BS, respectively. For convenience we assume that $P_1 > P_2$.¹ Each user chooses target power P_1 with probability p and P_2 with probability q , respectively. Using the channel inversion [2]–[4], each user takes the transmit power P_i/Y such that P_i can be the receive power at the BS. Notice that the channel reciprocity of TDD enables us to assume that the downlink channel gain Y would remain identical for a short period of time upon users' uplink packet transmissions.

For the decoding of the received packets at the BS, a packet with the target power P_1 is decoded successfully, if it is the only packet with P_1 and there is at most one packet with P_2 . In particular, the signal-to-interference noise ratio (SINR) of the packet targeting at P_1 should satisfy $\frac{P_1}{kP_2+N_0} \geq \nu$, where $k = \{0, 1\}$ is the number of users with target power P_2 and N_0 and ν denote the white noise power and the SINR threshold for the successful decoding, respectively. After decoding the packet with P_1 , the BS can also decode the packet with P_2 successfully by using SIC if it is the only received packet with P_2 and the signal-to-noise ratio (SNR) satisfies $\frac{P_2}{N_0} \geq \nu$.

¹In this letter, we exploit two different power levels as in the conventional studies [2]–[4].

When the minimum power is considered for P_1 and P_2 , we have $P_2 = N_0\nu$ and $P_1 = N_0\nu(1 + \nu)$.

Additionally, we assume that the users place a preamble in each packet, whose location depends on P_i , in order for the BS to estimate channel gain. We further assume perfect SIC at the BS. Just before the beginning of the next slot, the BS notifies the outcome of the packet transmissions, i.e., success, so that the users that do not receive the success feedback shall retransmit at the next slot.

B. Non-Cooperative NOMA RA Game

In this game, user i (or player i) for $i \in \mathcal{N} = \{1, 2, \dots, N\}$ has a strategy set $\mathcal{S}_i = \{H, L, W\}$, where H (or L) denotes action ‘transmit with target power P_1 (or P_2)’ respectively and W means ‘no transmission’ or wait. When both user 1 and 2 choose strategy H (or L), this results in a collision so that both get $-C_1$ (or $-C_2$). Due to channel inversion, the cost C_i is proportional to the transmit power, i.e., $C_i \propto P_i/Y$. If one chooses strategy H and the other does strategy L with no (re)transmission from other $N - 2$ users, they get a reward R as $R = \log(1 + \nu)$. Thus, the payoff that the user with strategy H (or L) is $R - C_1$ (or $R - C_2$). If user i chooses strategy W , he gets a cost $-\xi$, which is a cost of waiting. It is notable that introducing the cost of waiting forces users to (re)transmit as if increasing R encourages. However, ξ may take a different functional form.

To define this game formally, let us denote by \mathcal{S}_{-i} the strategy (vector) set of all other $N - 1$ users except user i , while s_i and \mathbf{s}_{-i} denote an element of \mathcal{S}_i and \mathcal{S}_{-i} . Furthermore, let $u_i(s_i, \mathbf{s}_{-i})$ be a payoff function of user i for strategic profile s_i and \mathbf{s}_{-i} . The NOMA RA game \mathcal{G} is defined as $\mathcal{G} = (\mathcal{N}, (\mathcal{S}_i)_{i \in \mathcal{N}}, (u_i)_{i \in \mathcal{N}})$. Since we consider a symmetric game \mathcal{G} , each user chooses strategy H with probability p , whereas strategy L with probability q . Therefore, strategy W is chosen with probability $r = 1 - p - q$.

Before introducing u_i , let us define $\beta(x)$ for $x = \{p, q\}$ as

$$\beta(x) = \sum_{k \in \{0,1\}} \binom{N-1}{k} x^k r^{N-1-k}. \quad (1)$$

If user i chooses strategy H , $\beta(q)$ accounts for the probability that he gets payoff $R - C_1$, while $N - 1$ users make no transmission, or only one of $N - 1$ users transmits with target power P_2 by choosing strategy L without any transmission from $N - 2$ users. Accordingly, $1 - \beta(q)$ accounts for the probability of the transmission failure of user i with strategy H . Upon strategy H , the payoff of user i is expressed as

$$u_i(H, \mathbf{s}_{-i,H}) = (R - C_1)\beta(q) - C_1(1 - \beta(q)) = R\beta(q) - C_1, \quad (2)$$

where $\mathbf{s}_{-i,H}$ denotes the strategy set of $N - 1$ users; that is, one user takes strategy L with $N - 2$ waiting users or no (re)transmission from $N - 1$ users. Similarly, his payoff upon strategy L is expressed as

$$\begin{aligned} u_i(L, \mathbf{s}_{-i,L}) &= (R - C_2)\beta(p) - C_2(1 - \beta(p)) \\ &= R\beta(p) - C_2, \end{aligned} \quad (3)$$

where $\mathbf{s}_{-i,L}$ indicates the strategy set that a user chooses strategy H with $N - 2$ waiting users or no (re)transmission from

$N - 1$ users. Particularly when he chooses strategy W , we assume that he gets a cost of waiting:

$$u_i(W, \mathbf{s}_{-i,W}) = -\xi, \quad (4)$$

which is independent of other users’ strategy, and $\mathbf{s}_{-i,W}$ denotes the entire strategy set of $N - 1$ users, i.e., $\mathbf{s}_{-i,W} \equiv \mathcal{S}_{-i}$. It is notable that if $C_1 = C_2 = C$, it implies that the system uses $P_1 = P_2 = P$ and users do not have any preference. Thus, we have S-ALOHA system. In the following section, we deal with $C_1 = C_2 = C$ as a special case.

III. ANALYSIS

Lemma 1: MNE of this symmetric NOMA RA game, denoted by (p^*, q^*, r^*) , can be obtained by solving $g(p) = 0$:

$$\begin{aligned} g(p) &= (1 + (N - 3)p)(\delta_2 - (\delta_1 + \delta_2)p)^{N-2} \\ &\quad - \frac{1}{R} \left(\delta_2 + \frac{\Delta C}{N-1} \right)^{N-1}, \end{aligned} \quad (5)$$

where $\Delta C = C_1 - C_2$, and $\delta_i = C_i - \xi$ for $i = \{1, 2\}$. The corresponding q^* is obtained as

$$q^* = \frac{\Delta C + [(N-1)\delta_1 - \Delta C]p^*}{(N-2)\delta_2 + \delta_1}. \quad (6)$$

Proof: To get an MNE, we apply indifference principle [8] as

$$u_i(H, \mathbf{s}_{-i}^\circ) = u_i(L, \mathbf{s}_{-i}^\bullet) \Rightarrow R(\beta(q) - \beta(p)) = \Delta C. \quad (7)$$

Using (1) and (7) with $r = 1 - p - q$, we have

$$(N-1)(q-p)r^{N-2} = \frac{\Delta C}{R}. \quad (8)$$

By using $u_i(H, \mathbf{s}_{-i}^\circ) = u_i(W, \bar{\mathbf{s}}_{-i})$, we can write

$$(1-p+(N-2)q)r^{N-2} = \frac{\delta_1}{R}. \quad (9)$$

By modifying (8) as $r^{N-2} = \frac{\Delta C}{R(N-1)(q-p)}$ and plugging it back into (9), we have (6). When substituting (6) into (8) and rearranging it with respect to p , we obtain (5). ■

As an alternative, we can find q^* first and then p^* . Using $u_i(L, \mathbf{s}_{-i}^\bullet) = u_i(W, \bar{\mathbf{s}}_{-i})$, we have $(1-q+(N-2)p)r^{N-2} = \frac{\delta_2}{R}$. When substituting $r^{N-2} = \frac{\Delta C}{R(N-1)(q-p)}$ into this, we get

$$p = \frac{(\Delta C + (N-1)\delta_2)q - \Delta C}{(N-1)\delta_2 + (N-2)\Delta C}. \quad (10)$$

Therefore, if q^* is found, then p^* is obtained with (10). Using (8) and (10), we get another polynomial in q as

$$(1 + (N-3)q)(\delta_1 + (\delta_1 + \delta_2)q)^{N-2} = \frac{1}{R} \left(\delta_1 - \frac{\Delta C}{N-1} \right)^{N-1},$$

whose solution is q^* .

Additionally, if $C_1 = C_2$, it can be found that q^* in (6) is reduced to $q^* = p^*$. This means that users choose strategy H , or L without any preference due to an equal cost.

Let us consider some bounds of p^* Lemma 2 and its positiveness in Lemma 3.

Lemma 2: The MNE p^* is bounded as

$$\frac{-\Delta C}{(N-1)\delta_1 - \Delta C} = pl \leq p^* \leq pu = \frac{\delta_2}{\delta_1 + \delta_2}. \quad (11)$$

Proof: Since $p^* + q^* \leq 1$, using (6) we write

$$p^* + q^* = p^* + \frac{\Delta C + [(N-1)\delta_1 - \Delta C]p^*}{(N-2)\delta_2 + \delta_1} \leq 1. \quad (12)$$

Rearranging (12) with respect to p^* is reduced to p_u . We get p_l by imposing $q^* \geq 0$ on (6). ■

Lemma 3: In the game \mathcal{G} , for positive p^* and q^* , the system should have $\delta_1 > 0$ and $\delta_2 > 0$ (i.e., $C_1 > \xi$, $C_2 > \xi$) (the system with the transmission costs larger than the cost of waiting), or $\delta_1 < 0$ and $\delta_2 < 0$ (the system with the transmission costs smaller than the cost of waiting).

Proof: When imposing the condition $q^* \leq 1$ on (6), we can rewrite (6) with respect to p^* as

$$p^* \leq \delta_2(\delta_1 - \Delta C/(N-1))^{-1}. \quad (13)$$

As $N \rightarrow \infty$, we can expect that $p^* \leq \frac{\delta_2}{\delta_1 - \epsilon}$ for a small positive ϵ . For p^* and q^* to be positive, we should have either $\delta_1 > 0$ and $\delta_2 > 0$, or $\delta_1 < 0$ and $\delta_2 < 0$. ■

It is notable that Lemma 3 guarantees the positiveness of p_u . If $C_1 = C_2$, Lemma 2 and 3 give that $p_l = 0 \leq p^* \leq p_u = \frac{1}{2}$. In what follows, we thus characterize the solution of (5) for $\delta_1 > 0$ and $\delta_2 > 0$ in Theorem 1 and for $\delta_1 < 0$ and $\delta_2 < 0$ in Theorem 2, respectively.

Theorem 1: For $C_1 > \xi$ and $C_2 > \xi$, the solution of (5), i.e., $p^* \in (0, 0.5)$, is the unique MNE, if

$$R\delta_2^{N-2} > (\delta_1 + \Delta C/(N-1))^{N-1}. \quad (14)$$

Proof: We consider when N is odd or even. If N is odd, $g(p)$ at $p = 1$ is obtained as

$$g(1) = (N-2)(-\delta_1)^{N-2} - \frac{(\delta_2 + \Delta C/(N-1))^{N-1}}{R}. \quad (15)$$

Since $C_1 > \xi$ ($\delta_1 > 0$), $g(1) < 0$ is always found for odd N .

Let us examine whether $g(p)$ is a decreasing function of p , or not. To do this, let us find

$$\begin{aligned} \frac{dg(p)}{dp} &= -(\delta_2 - (\delta_1 + \delta_2)p)^{N-3} \\ &\times \left[\delta_2 + (N-2)\delta_1 + p(\delta_1 + \delta_2)(N^2 - 4N + 6) \right]. \end{aligned} \quad (16)$$

The term inside the square brackets in (16) is positive by the assumption $C_1 > \xi$ and $C_2 > \xi$ ($\delta_1 > 0$ and $\delta_2 > 0$). When N is odd, (16) is always negative. Since $g(p)$ decreases in p , it has a unique p^* if $g(0) > 0$. We have $g(0)$ as

$$g(0) = \delta_2^{N-2} - \frac{1}{R}(\delta_2 + \Delta C/(N-1))^{N-1}. \quad (17)$$

Setting $g(0) > 0$ yields (14) and we can conclude that $g(p)$ has a single solution. Note that in order to have $q^* < 1$ in (6), we should have (13). From $\frac{dg(p)}{dp} = 0$ in (16), we can see that $g(p)$ has two stationary points: $p_1 = \frac{\delta_2}{\delta_1 + \delta_2}$, and $p_2 = -\frac{\delta_2 + (N-2)\delta_1}{(\delta_1 + \delta_2)(N^2 - 2N + 6)}$. Since p_2 is negative due to $C_1 > \xi$ and $C_2 > \xi$, it is not considered any more. Comparing p_1 with p_u , we can see that $p_1 = p_u$. Thus, the unique solution of $g(p)$, i.e., the MNE, is less than p_1 . As a special case, if $C_1 = C_2$, it follows that $p_1 = \frac{1}{2}$.

Let us consider when N is even. Note that $g(p)$ is decreasing for $0 \leq p < p_1$ in (16). If we have

$$g(p_1) = -\frac{1}{R}(\delta_2 + \Delta C/(N-1))^{N-1} < 0, \quad (18)$$

then $g(p)$ has a unique solution when N is even. Since the exponent $N-1$ in (18) is odd for even N , to have $g(p_1) < 0$ we should have

$$\delta_2 + \Delta C/(N-1) > 0 \Rightarrow \Delta C > -(N-1)\delta_2. \quad (19)$$

By our assumption on $C_2 > \xi$, (19) holds for some large N , i.e., $N > 1 - \frac{\Delta C}{\delta_2}$. As long as $g(0) > 0$ holds, it follows that the game admits a unique MNE. This completes the proof. ■

Corollary 1: When $C_1 = C_2 = C$ and $C > \xi$, the solution of (5), i.e., $p^* \in (0, 0.5)$, is the unique MNE, if $R > C - \xi$.

Proof: If $C_1 = C_2 = C$, (15) is reduced to $g(1) = (N-2)(\xi - C)^{N-2} - \frac{1}{R}(C - \xi)^{N-1}$. If N is odd, it follows that $g(1) < 0$ and the condition $g(0) > 0$ in (17) becomes the condition $R > C - \xi$. Since $g(p_1) = -\frac{1}{R}(C - \xi)^{N-1} < 0$, it follows that $g(p)$ admits only one MNE. If N is even, (19) always holds. Thus, if $R > C - \xi$ holds, which provides that $g(0) > 0$, there is only one MNE. ■

Theorem 2: When $C_1 < \xi$ and $C_2 < \xi$, there is no MNE.

Proof: If N is odd, it is found that $g(0) < 0$ in (17) due to $\delta_2 < 0$ ($C_2 < \xi$). Furthermore, since $g(p_1) < 0$, i.e., the minimum of $g(p)$, and $g(1) > 0$ for $C_1 < \xi$ and $C_2 < \xi$, it is expected that the solution of $g(p)$ is greater than p_1 . However, owing to Lemma 2, this solution can not be the MNE.

Let us consider when N is even. In this case, we can see that $g(0) > 0$. If we show that $g(p_1) > 0$, there is no MNE, which is less than 0.5. For $g(p_1)$ to be positive in (18), since the exponent $N-1$ in (18) is odd, we need to have

$$\delta_2 + \Delta C/(N-1) < 0, \quad (20)$$

which is rewritten as $(N-1)\delta_2 < -\Delta C$. In most of cases, (20) is true, since $\delta_2 < 0$ ($C_2 < \xi$) as N grows. Notice that for $C_1 = C_2 = C$, (20) always holds since $C - \xi < 0$, which implies no MNE. ■

The following corollary characterizes NE of two-user game.

Corollary 2: If $N = 2$ (two-user game), its mixed strategy NE is obtained as $p^* = \frac{R-\delta_1}{R}$ and $q^* = \frac{R-\delta_2}{R}$.

Proof: Substituting $N = 2$ into (5) yields the result. In order to have $0 \leq p^* + q^* \leq 1$, we should have $\max(\delta_1, \delta_2) \leq R \leq \delta_1 + \delta_2$. Thus, R is positive if $C_1 > \xi$ and $C_2 > \xi$. ■

Theorem 1 and 2 show that in uplink NOMA RA systems, if the transmission cost C_1 and C_2 are respectively smaller than the cost of waiting ξ , there is no MNE, possibly due to excessive collisions.

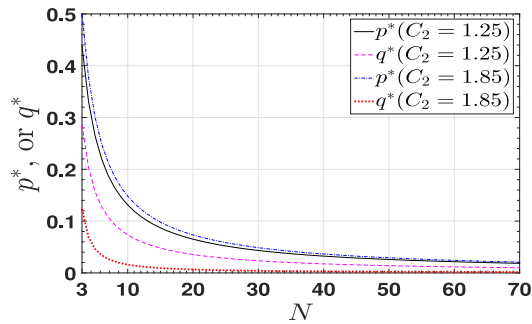
Finally, let us examine the efficiency of MNE with respect to SWM. In SWM, the system maximizes the sum of the expected payoff of all users, which is defined as $S_o(p, q, r) \triangleq \sum_{i=1}^N u_i(H, s_{-i,H})p + u_i(L, s_{-i,L})q + u_i(W, s_{-i,W})r$. Then, SWM can be formulated as

$$\begin{aligned} &\underset{p, q, r}{\text{maximize}} && S_o(p, q, r) \\ &\text{subject to} && p + q + r = 1 \\ &&& 0 \leq p, q, r \leq 1, \end{aligned} \quad (21)$$

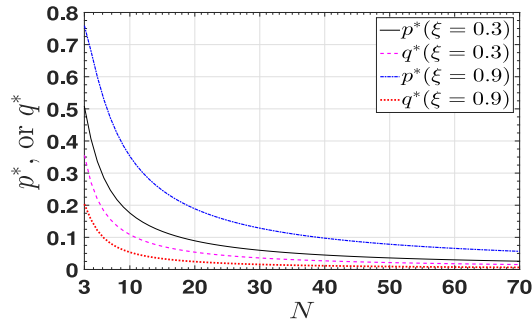
which can be numerically solved.

IV. NUMERICAL STUDIES

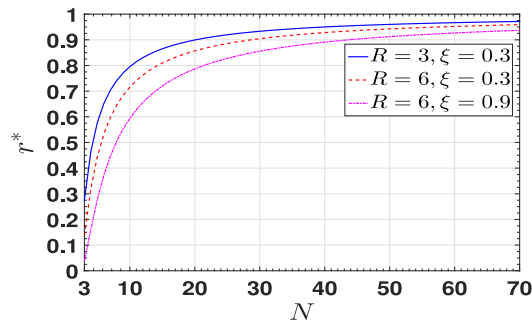
Figs. 1(a)-1(c) illustrate MNE (p^* , q^* , r^*) according to the parameters R , C_1 , C_2 , and ξ , while Fig. 2 depicts SWM.



(a)



(b)



(c)

Fig. 1. MNE with various rewards and costs. (a) MNE with $R = 3$, $\xi = 0.3$ and $C_1 = 1$ (b) MNE with $R = 6$, $C_1 = 1$ and $C_2 = 1.25$ (c) MNE with $C_1 = 1$ and $C_2 = 1.25$.

In Fig. 1(a) we set $R = 3$, $C_1 = 1$ and $\xi = 0.3$, while increasing N and C_2 . As N increases (contention increases), it can be seen that both p^* and q^* decrease. In other words, users prefer waiting to (re)transmission. When C_2 increases, the probability that users choose strategy H , i.e., p^* , slightly increases, while q^* significantly decreases due to a higher C_2 . If C_2 would be even more increased, it is expected that $q^* \rightarrow 0$. In addition, when we set $C_1 = 1.25$ and $C_2 = 1$, the previous p^* with $C_1 = 1$ and $C_2 = 1.25$ becomes q^* , whereas q^* becomes p^* .

In Fig. 1(b) the reward R and/or the cost of waiting ξ is increased compared to Fig. 1(a). As it might be expected, p^* and q^* increase with an increased R . In particular, when the cost of waiting increases close to C_1 , user's preference to strategy H increases, while strategy L is much less preferred.

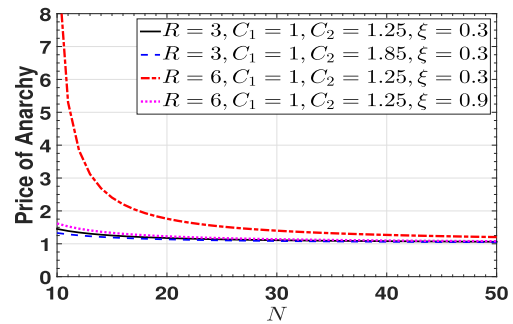


Fig. 2. Social welfare maximization.

Fig. 1(c) shows the MNE of strategy W , i.e., $r^* = 1 - p^* - q^*$. Upon a higher R and/or ξ , it is found that the decision W is less preferred, since users are much more motivated to make (re)transmissions. Fig. 2 depicts the price of anarchy (PoA), which is the ratio of the expected user payoff with MNE to that with SWM. Note that since our symmetric non-cooperative game has a unique MNE, the expected payoff that users obtain with the MNE is $-\xi$ by the indifference principle. In finding PoA, we normalize the user payoff with SWM by N , so that it represents one user payoff with SWM. The PoA shows how much the efficiency (or SW) can degrade as the number of selfish users increases. It can be seen that as R (or C_i) increases (or decreases), the PoA increases.

V. CONCLUSION

This letter has investigated N -user NOMA RA systems with non-cooperative game by taking into account the cost of waiting. We have shown that if the cost of waiting is smaller than the transmission cost, the game admits a unique MNE; otherwise, no MNE. Numerical results have shown that as the number of users increases and/or the cost of waiting decreases, the users are less willing to (re)transmit. As future work, we are interested in developing an access algorithm based on multiagent reinforcement learning in conjunction with the non-cooperative game presented here.

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