A Pre-Whitening Scheme in a MIMO-Based Spectrum-Sharing Environment

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***Abstract*—In this letter, we propose a pre-whitening scheme for a spectrum-sharing environment where multiple antennas are used for both primary and secondary users. The proposed pre-whitening scheme is different from the conventional post-whitening scheme for mitigating multiple-input and multiple-output (MIMO) interference because a primary receiver does not require any pre-knowledge about the interference channel which is a more proper assumption in a spectrum-sharing environment. Moreover, the proposed scheme outperforms the conventional post-whitening scheme in terms of the secondary user capacity due to its interference-reduction capability.**

***Index Terms*—Cognitive radio, spectrum sharing, MIMO, interference.**

I. INTRODUCTION

As the required radio spectrum is growing rapidly, *Cognitive Radio* (CR) [1] has been an attractive solution for efficiently utilizing the scarce radio spectrum. The concept of *interference temperature* was first proposed by the Federal Communication Commission in 2002 [2], and it inspired a study called *Spectrum-sharing* under interference temperature at the licensed users, which enables unlicensed users to share the licensed spectrum. Spectrum-sharing was first studied under an additive white Gaussian noise (AWGN) environment [3]. Later, Ghasemi and Sousa [4] studied the achievable rate of a secondary user under the assumptions: (i) no interference is generated from the primary user; and (ii) wireless channels are independently Rayleigh-distributed. They showed that the opportunity obtained from the fluctuating interference channel increases the channel capacity of the secondary user. Zhang and Liang [5] extended the spectrum-sharing model in a MIMO configuration and developed optimal and sub-optimal transmit covariance matrices at the secondary transmitter to maximize the capacity of the secondary users under interference constraints at the primary receivers.

Several papers [4] [5] only set a received interference power constraint at a primary receiver and found an optimal transmit scheme to maximize the capacity of secondary users. However, as the transmitters and the receivers are equipped with multiple antennas, many factors such as the received interference power, the received interference signal structure, and interference mitigation schemes affect the performance of the primary and the secondary users. Considering these features, in this letter, we propose a *pre-whitening* scheme to share the spectrum more efficiently in a MIMO environment and reduce the complexity at the primary receiver.

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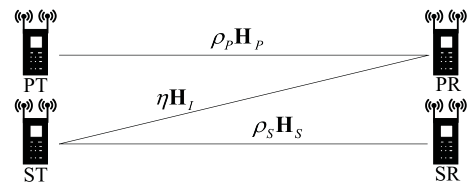

Fig. 1. System Model for MIMO Spectrum-Sharing Environment

II. SYSTEM MODEL OF MIMO BASED SPECTRUM-SHARING

Fig. 1 shows a system model of a MIMO-based spectrum-sharing environment. As described in previous spectrum-sharing studies [4] [5], we assume that a secondary transmitter (ST) induces interference at a primary receiver (PR), while a primary transmitter (PT) does not give any interference to a secondary receiver (SR).

The received signal vectors at the primary and secondary receivers, $\mathbf{y}_P$ and $\mathbf{y}_S$, are given, respectively, as

$$\mathbf{y}_P = \sqrt{\rho_P}\sqrt{P_{PT}}\mathbf{H}_P\mathbf{x}_P + \sqrt{\eta}\sqrt{P_{ST}}\mathbf{H}_I\mathbf{x}_S + \mathbf{w}_P, \quad (1)$$

$$\mathbf{y}_S = \sqrt{\rho_S}\sqrt{P_{ST}}\mathbf{H}_S\mathbf{x}_S + \mathbf{w}_S, \quad (2)$$

where P_{PT} (P_{ST}) is the transmit power and $\mathbf{x}_P$ ($\mathbf{x}_S$) is the normalized transmitted signal vector, i.e., $\mathbb{E}[\|\mathbf{x}_P\|^2] = 1$ ($\mathbb{E}[\|\mathbf{x}_S\|^2] = 1$), at the primary (secondary) transmitter. P_{ST} is assumed to be not limited to a certain value. The terms $\mathbf{w}_P$ and $\mathbf{w}_S$ are $N \times 1$ AWGN vectors with variance of $N_0/2$ for each dimension. The channel matrices $\mathbf{H}_P$, $\mathbf{H}_S$ and $\mathbf{H}_I$ represent the channel between the PT and PR, the ST and SR, and, the ST and the PR, respectively. They all are $N \times N$ matrices with complex entries which independently follow $CN(0, 1)$, where all users are assumed to have the same number of transmit and receive antennas N for notational simplicity. The results can be extended to a more general case with a different number of antennas. The terms ρ_P, ρ_S, and η are the average channel gains for $\mathbf{H}_P$, $\mathbf{H}_S$, and $\mathbf{H}_I$, respectively. We assume that the primary and the secondary transmitters do not have the channel information $\mathbf{H}_P$ and $\mathbf{H}_S$, respectively, but channel state information at the receiver (CSIR) is available. On the other hand, the secondary transmitter has $\mathbf{H}_I$ in advance by scanning the interference channel.

III. INTERFERENCE MITIGATION SCHEMES FOR MIMO SPECTRUM-SHARING

A. Conventional Post-Whitening Scheme

First, we consider a conventional MIMO interference management technique, called a post-whitening scheme. The V-BLAST (Vertical Bell Labs Space-Time) with equal power allocation to each transmit antenna, which is known as the optimal MIMO transmit scheme under the assumption of CSIR, is used in the primary and the secondary users. We define an information signal vector $\mathbf{u}_P$ of $N \times 1$ which has N independent Gaussian data streams with $\mathbb{E}[\mathbf{u}_P \mathbf{u}_P^H] = \mathbf{I}_N$ and the normalized signal vector $\mathbf{x}_P = \frac{\mathbf{u}_P}{\|\mathbf{u}_P\|}$. Thus, the covariance matrix for the primary transmitter is given by $\mathbf{K}_P = \frac{P_{PT}}{N}\mathbf{I}_N$. Signal vectors $\mathbf{u}_S$ and $\mathbf{x}_S$ and a covariance matrix $\mathbf{K}_S$ for the secondary transmitter are defined in a similar way.

The interference received at the primary receiver is spatially correlated over multiple receive antennas. To decode its own signals, the post-whitening scheme that uncorrelates the spatially correlated interference is optimal among linear methods. To utilize it, the primary receiver should have interference channel matrix $\mathbf{H}_I$ in advance. The interference plus noise vector of the primary receiver, $\mathbf{z}$, is defined as $\mathbf{z} = \sqrt{\eta}\sqrt{P_{ST}}\mathbf{H}_I\mathbf{x}_S + \mathbf{w}_P$, where the covariance matrix of $\mathbf{z}$ is given by

$$\mathbf{R} \triangleq \mathbb{E}[\mathbf{z}\mathbf{z}^H] = \frac{\eta P_{ST}}{N}\mathbf{H}_I\mathbf{H}_I^H + N_0\mathbf{I}_N. \quad (3)$$

To uncorrelate (whiten) $\mathbf{z}$, $\mathbf{R}^{-1/2}$ is multiplied to $\mathbf{y}_P$. Finally, the MIMO channel capacity of the primary user after whitening the interference is given by [6]

$$\begin{aligned} C_P &= \log_2 \det\left(\mathbf{I}_N + \rho_P \mathbf{R}^{-1/2}\mathbf{H}_P\mathbf{K}_P\mathbf{H}_P^H\mathbf{R}^{-1/2}\right) \quad (4)\\ &= \log_2 \det\left(\mathbf{I}_N + \frac{\rho_P P_{PT}}{N}\mathbf{H}_P\mathbf{H}_P^H\mathbf{R}^{-1}\right), \quad (5)\end{aligned}$$

where Eq. (5) is obtained from Eq. (4) by using the following relation; $\det(\mathbf{I}_N + \mathbf{AB}) = \det(\mathbf{I}_N + \mathbf{BA})$.

In the mean time, the capacity for the secondary users is given by

$$C_S = \log_2 \det\left(\mathbf{I}_N + \frac{\rho_S P_{ST}}{N}\frac{\mathbf{H}_S\mathbf{H}_S^H}{N_0}\right). \quad (6)$$

B. Proposed Pre-Whitening scheme

As previously noted, the conventional post-whitening scheme requires that the primary receiver must keep track of the interference channel, $\mathbf{H}_I$, all the time. However, it violates the philosophy of cognitive radios that the secondary networks should be transparent to the primary user. We propose a *pre-whitening* scheme to achieve efficient spectrum-sharing among the primary and secondary users when the primary receiver does not have the interference channel information. Instead of whitening the spatially correlated interference at the primary receiver as in the conventional post-whitening scheme, the pre-whitening scheme whitens the correlated interference *at the secondary transmitter*. The proposed pre-whitening scheme assumes that the interference channel $\mathbf{H}_I$ is perfectly known to the secondary transmitter so that the primary receiver does not need the knowledge of interference channel, which agrees with the philosophy of cognitive radios. Since we assume that the primary transmitter sends its data using V-BLAST scheme, the normalized signal for the primary transmitter, $\mathbf{x}_P$, is defined as in the conventional post-whitening scheme. On the other hand, the normalized signal for the secondary transmitter is defined as $\mathbf{x}_S = \frac{\mathbf{H}_I^{-1}\mathbf{u}_S}{\sqrt{\gamma}}$. Note that the inverse of the interference channel matrix $\mathbf{H}_I$ is multiplied to $\mathbf{u}_S$ so that the interference received at the primary receiver is spatially uncorrelated. The computational error of calculating the inversion of $\mathbf{H}_I$ is assumed to be ignored in this letter, since we assume a rich scattering environment for MIMO channels. The normalizing term γ is defined as following:

$$\begin{aligned}\gamma &\triangleq \mathbb{E}\left[\|\mathbf{H}_I^{-1}\mathbf{u}_S\|^2\right] = \text{trace}\left(\mathbf{H}_I^{-1}\mathbb{E}\left[\mathbf{u}_S\mathbf{u}_S^H\right]\left(\mathbf{H}_I^{-1}\right)^H\right) \quad (7)\\ &= \text{trace}\left(\left(\mathbf{H}_I^H\mathbf{H}_I\right)^{-1}\right). \quad (8)\end{aligned}$$

Moreover, the covariance matrix of the secondary transmitter is given as

$$\mathbf{K}_S = \frac{P_{ST}}{\gamma}\mathbb{E}\left[\mathbf{H}_I^{-1}\mathbf{u}_S\mathbf{u}_S^H\left(\mathbf{H}_I^{-1}\right)^H\right] = \frac{P_{ST}}{\gamma}\left(\mathbf{H}_I^H\mathbf{H}_I\right)^{-1}. \quad (9)$$

When the pre-whitening scheme is applied, the received signal given in Eq. (1) becomes

$$\mathbf{y}_P = \sqrt{\rho_P}\sqrt{P_{PT}}\mathbf{H}_P\mathbf{x}_P + \frac{\sqrt{\eta}\sqrt{P_{ST}}}{\sqrt{\gamma}}\mathbf{u}_S + \mathbf{w}_P. \quad (10)$$

Note that the interference vector is a scalar-multiplied Gaussian signal vector with N independent data streams. The interference signal vector and the noise signal vector have the same statistical characteristics so that the two terms can be represented as a single random vector whose covariance matrix is $\left(\frac{\eta P_{ST}}{\gamma} + N_0\right)\mathbf{I}_N$.

As a result, the capacities of the primary and the secondary users are expressed as

$$\begin{aligned} C_P &= \log_2 \det\left(\mathbf{I} + \frac{\rho_P P_{PT}}{N}\frac{\mathbf{H}_P\mathbf{H}_P^H}{\left(\frac{\eta P_{ST}}{\gamma} + N_0\right)}\right), \quad (11)\\ C_S &= \log_2 \det\left(\mathbf{I} + \frac{\rho_S P_{ST}}{\gamma}\frac{\mathbf{H}_S\left(\mathbf{H}_I^H\mathbf{H}_I\right)^{-1}\mathbf{H}_S^H}{N_0}\right). \quad (12)\end{aligned}$$

C. Mathematical Comparison between the Post-Whitening and the Pre-Whitening Schemes

In this subsection, we mathematically analyze the performance of both the post-whitening and the pre-whitening schemes.

For mathematical analysis, we obtain the following theorem which examines the difference between the post-whitening scheme and the pre-whitening scheme in terms of the received interference power at the primary receiver.

Theorem 1: If the secondary transmitter uses the same transmit power P_{ST} for the post-whitening and the pre-whitening schemes, the inequality $P_{\text{int,PoW}} \geq P_{\text{int,PrW}}$ always holds with equality iff $\mathbf{H}_I$ has N identical eigenvalues, where $P_{\text{int,PoW}}$ and $P_{\text{int,PrW}}$ represent the received interference power at the primary receiver when the post-whitening scheme and the pre-whitening scheme are used, respectively.

Proof: See Appendix. ■

From Theorem 1, the pre-whitening scheme outperforms the post-whitening scheme in terms of the received interference power at the primary receiver because the received interference power at the primary receiver for the pre-whitening scheme is less than or equal to that of the post-whitening scheme.

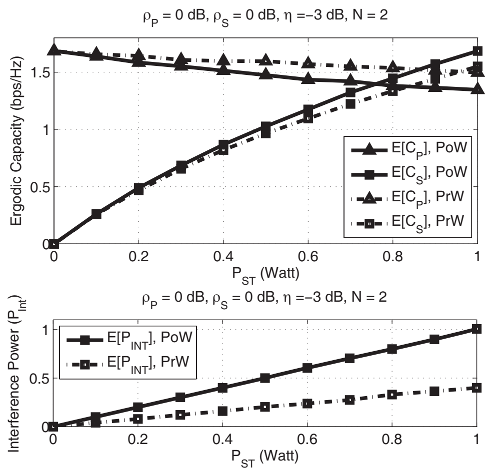

Fig. 2. Ergodic Capacities and Received Interference Power for the post-whitening scheme and the Pre-whitening scheme when we vary P_{ST}.

IV. Numerical Examples

Fig. 2 shows the ergodic capacities and received interference power for the post-whitening scheme and the pre-whitening scheme when we vary P_{ST}. The amount of received interference power of the pre-whitening scheme is considerably smaller than that of the post-whitening scheme. This result agrees with that of Theorem 1. On the other hand, the secondary user capacity of the pre-whitening scheme is slightly less than that of the post-whitening scheme since the secondary transmitter of the pre-whitening scheme does not use the optimal identity matrix for its covariance matrix. However, this degradation due to the non-optimal covariance matrix can be easily compensated because the secondary transmitter with the pre-whitening scheme can use more transmit power than that with the post-whitening scheme to affect the same received interference power at the primary receiver.

Finally, we observe the performance of the two schemes when there is a received interference power constraint at the primary receiver. If this constraint Q is set at the primary receiver, the transmit power at the secondary transmitter should be controlled so that the received interference power is less than Q. Fig. 3 shows the ergodic capacities of the two schemes for varying Q. In the figure, the ergodic capacities of the primary users for both the post-whitening and the pre-whitening schemes are almost the same, while the ergodic capacity of the secondary users for the pre-whitening scheme is significantly higher than for the post-whitening scheme for the whole range of Q. Since the pre-whitening scheme causes less interference to the primary receiver than the post-whitening scheme, the secondary transmitter can use more transmit power to meet the constraint Q. Due to the boosted transmit power of the secondary transmitter, the pre-whitening scheme outperforms the post-whitening scheme.

V. Conclusion

We proposed a pre-whitening scheme in a MIMO spectrum-sharing environment and compared the performance of the proposed scheme with that of the conventional post-whitening scheme. The pre-whitening scheme does not require any pre-knowledge about the interference channel at the primary receiver, which is a more practical assumption in a spectrum-sharing environment. Our analysis and numerical results showed that the proposed pre-whitening scheme outperforms

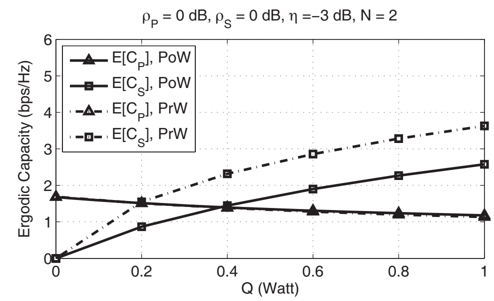

Fig. 3. Ergodic Capacities of the Post-Whitening scheme and the Pre-Whitening scheme for Varying Q.

the conventional post-whitening scheme in terms of the primary user capacity at the cost of the secondary user capacity. However, the reduction of the secondary user capacity can be dealt with an increase of secondary transmitter's power if the transmit power budget of the secondary transmitter is available. The proposed pre-whitening scheme can be extended to multiple secondary transmitters even though this letter considered a single secondary transmitter, which remains as our further study.

Appendix
Proof of Theorem 1

From Eqs. (1), (10), and (8), $P_{\text{int,PoW}}$ and $P_{\text{int,PrW}}$ are given by

$$P_{\text{int,PoW}} = \eta P_{ST} \frac{\text{trace}\left(\mathbf{H}_I \mathbf{H}_I^H\right)}{N}. \quad (13)$$

$$P_{\text{int,PrW}} = \eta P_{ST} \frac{N}{\gamma} = \eta P_{ST} \frac{N}{\text{trace}\left(\left(\mathbf{H}_I^H \mathbf{H}_I\right)^{-1}\right)}. \quad (14)$$

We define the singular values of $\mathbf{H}_I$ as $\sqrt{\lambda_1}, \sqrt{\lambda_2}, \ldots, \sqrt{\lambda_N}$. From $\text{trace}\left(\mathbf{H}_I \mathbf{H}_I^H\right) = \sum_{i=1}^{N} \lambda_i$ and $\text{trace}\left(\left(\mathbf{H}_I^H \mathbf{H}_I\right)^{-1}\right) = \sum_{i=1}^{N} \frac{1}{\lambda_i}$, the received interference power at the primary receiver for the post-whitening and the pre-whitening schemes are given, respectively, by

$$P_{\text{int,PoW}} = \eta P_{ST} \left(\left(\sum_{i=1}^{N} \lambda_i\right) / N\right) = \eta P_{ST} \cdot \mathrm{A}(\lambda_1, \ldots, \lambda_N) \quad (15)$$

$$P_{\text{int,PrW}} = \eta P_{ST} \left(N / \left(\sum_{i=1}^{N} \frac{1}{\lambda_i}\right)\right) = \eta P_{ST} \cdot \mathrm{H}(\lambda_1, \ldots, \lambda_N), \quad (16)$$

where $\mathrm{A}(\lambda_1, \ldots, \lambda_N)$ and $\mathrm{H}(\lambda_1, \ldots, \lambda_N)$ denote the arithmetic mean and harmonic mean of λ_i's. From the fact that $\mathrm{A}(\lambda_1, \ldots, \lambda_N) \geq \mathrm{H}(\lambda_1, \ldots, \lambda_N)$, with equality iff all the λ_i's $(i = 1, \ldots, N)$ are the same, we conclude $P_{\text{int,PoW}} \geq P_{\text{int,PrW}}$.

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