Opportunistic Interference Alignment for MIMO Interfering Broadcast Channels

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***Abstract*—In this paper, we propose an opportunistic interference alignment (OIA) technique for cellular downlink networks, which efficiently reduces the effect of inter-cell interference from base stations (BSs) in other cells and eliminates intra-cell interference among spatial streams in the same cell. We show that the user scaling per cell required to achieve a target degrees-of-freedom can be fundamentally lowered, compared with the previous results. In addition, we relate the derived user scaling law to the interference decaying rate with respect to the number of users for given signal-to-noise ratio. Simulation results show that the proposed OIA significantly outperforms the previous schemes in terms of both sum-interference and achievable sum-rate even in practical environments.**

***Index Terms*—Degrees-of-freedom (DoF), opportunistic interference alignment (OIA), MIMO interfering broadcast channel (MIMO-IBC), transmit & receive beamforming, user scheduling.**

I. Introduction

Interference management is one of the most challenging issues to improve a cell throughput in cellular networks. It was shown that the interference alignment (IA) technique achieves the optimal degrees-of-freedom (DoF) in the K-user interference channel with time-varying channel coefficients [1]. Subsequent works have shown that the IA is also useful for other wireless networks including multiple-input multiple-output (MIMO) interference channels [2]–[4] and cellular networks [5]–[7].

On the other hand, there have been some notable techniques that exploit the benefit of fading in a single cell network, obtaining *multiuser diversity (MUD)* gain: opportunistic scheduling [8], opportunistic beamforming [9], and random beamforming [10]. Moreover, scenarios with achievable MUD gain have been studied in ad hoc networks [11], cognitive radio networks [12], and multi-cell downlink and uplink networks [13], [14].

Recently, an opportunistic interference alignment (OIA) concept which combines the IA and user scheduling was proposed for interfering multiple access channels (IMAC) [15]–[17]. OIA has been known to achieve the optimal DoF in IMAC if a certain user scaling condition is satisfied even though it operates in a distributed fashion. For multi-cell downlink networks, so called interfering broadcast channel (IBC), similar techniques were also proposed [13], [18]–[21]. In [13], it was shown that the optimal DoF of KM can be achieved if $N = \omega\left(\mathsf{SNR}^{KM-1}\right)$[1], where N, K, and M denote the number of users in a cell, total number of cells in the network, and number of transmit antennas at each BS, respectively. The authors extended the random beamforming technique, originally proposed for a single cell network in [10], to a multi-cell downlink assuming a single antenna at users. The authors of [19] obtained the same user scaling law as in [13] in the same network by using the same technique but using different derivations. In [21], the authors also considered the effect of multiple antennas at users on the required user scaling for the optimal DoF, i.e., $N = \omega\left(\mathsf{SNR}^{KM-L}\right)$ where L denotes the number of receive antennas at users. In [18], the user scaling for given DoF in a 3-cell single-input multi-output (SIMO) downlink network is derived. In the same work, a general K-cell downlink network and multiple antennas at BSs in [20] are also taken into account. The user scaling in [20] is the same as [21], since all of these previous works are based on the multi-cell random beamforming technique.

In this paper, we propose a novel OIA technique for MIMO cellular downlink networks, which efficiently reduces the effect of inter-cell interference from BSs in other cells and eliminates intra-cell interference due to the spatial streams dedicated to the other users in the same cell. In the proposed OIA, two cascaded precoders are used at the BSs similar to the scheme proposed in [6]. The first precoder eliminates the intra-cell interference due to the other selected users in the same cell. The second precoder plays the same role of multi-cell random beamforming. Specifically, it enables users to exactly estimate the interference subspace from the BSs. The receive beamforming vector is designed at each user using *local* channel state information (CSI) in a *distributed* manner, and each user feeds back the effective channel vector and quantity of inter-cell interference to the corresponding BS. The user selection at the BSs and design of receive beamforming vector are completely decoupled, and hence no iterative optimization as in [6] is needed.

We show that the user scaling required to achieve the optimal DoF of KM can be reduced to $N = \omega\left(\mathsf{SNR}^{(K-1)M-L+1}\right)$. In addition, the interference decaying rate with respect to N for given SNR is characterized in conjunction with the derived user scaling law. Furthermore, simulation results show that the proposed OIA significantly outperforms the previous schemes even in practical environments.

II. System and Channel Models

We consider K-cell MIMO IBC where each cell consists of a BS with M antennas and N users, each with L antennas. The number of users selected to receive downlink signals in each cell is denoted by $S \leq M$. It is assumed that each selected user receives a single spatial stream. To consider nontrivial cases, we assume that $L < (K-1)S + 1$, because all the inter-cell interference can be completely canceled at the receivers otherwise. The channel matrix from the k-th BS to the j-th user in the i-th cell is denoted by $\mathbf{H}_k^{[i,j]} \in \mathbb{C}^{L\times M}$, where $i,k \in \mathcal{K} \triangleq \{1,\ldots,K\}$ and $j \in \mathcal{N} \triangleq \{1,\ldots,N\}$. Each element of $\mathbf{H}_k^{[i,j]}$ is assumed to be independent and identically distributed (i.i.d.) according to $\mathcal{CN}(0,1)$. In addition, for given transmission block, quasi-static frequency-flat fading is assumed, i.e., channel coefficients are constant during the transmission block. From pilot signals sent from all the BSs, the j-th user in the i-th cell can estimate the channels $\mathbf{H}_k^{[i,j]}$, $k = 1,\ldots,K$, i.e., the local CSI at the transmitter.

[1] $f(x) = \omega(g(x))$ implies that $\lim_{x\to\infty} \frac{g(x)}{f(x)} = 0$.

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Without loss of generality, the indices of selected users in every cell are assumed to be $(1, \ldots, S)$. The total DoF is defined by

$$\text{DoF} = \lim_{\text{SNR}\to\infty} \frac{\sum_{i=1}^{K}\sum_{j=1}^{S} R^{[i,j]}}{\log \text{SNR}}, \tag{1}$$

where $R^{[i,j]}$ is the achievable rate for the j-th user in the i-th cell.

III. Proposed OIA for MIMO IBC

A. Overall Procedure

1) Initialization (Reference Precoding Matrix Broadcast): The predetermined reference precoding matrix of the k-th cell is denoted by $\mathbf{P}_k = [\mathbf{p}_{1,k}, \ldots, \mathbf{p}_{S,k}]$, where $\mathbf{p}_{s,k} \in \mathbb{C}^{M\times 1}$ is the orthonormal basis, $k \in \mathcal{K}$, $s = 1, \ldots, S$. The k-th BS independently generates $\mathbf{p}_{k,s}$ from the isotropic distribution over the M-dimensional unit sphere. Each user can estimate the effective channel $\mathbf{H}_k^{[i,j]}\mathbf{P}_k$ if the pilots are rotated by $\mathbf{P}_k$. The reference precoding matrix $\mathbf{P}_k$ can be regarded as cell-coordination, since it is determined prior to the user scheduling or data transmission. As explained later, in advance to the reference precoding $\mathbf{P}_k$, user-specific beamforming $\mathbf{V}_k$ is applied in the k-th cell, but it does not change the interference structure at users.

2) Receive Beamforming & Scheduling Metric Feedback: Let us define the unit-norm weight vector at the j-th user in the i-th cell by $\mathbf{u}_{[i,j]} \in \mathbb{C}^{L\times 1}$, i.e., $\left\|\mathbf{u}_{[i,j]}\right\|^2 = 1$. How to design $\mathbf{u}_{[i,j]}$ shall be presented in Section IV along with the corresponding user scaling law. From the notion of $\mathbf{P}_k$ and $\mathbf{H}_k^{[i,j]}$, the scheduling metric of the j-th user in the i-th cell, denoted by $\eta^{[i,j]}$, is defined by the sum of the received interference power from other cells. That is,

$$\eta^{[i,j]} = \sum_{k=1,k\neq i}^{K} \left\|\mathbf{u}_{[i,j]}{}^{*}\mathbf{H}_k^{[i,j]}\mathbf{P}_k\right\|^2. \tag{2}$$

All the users report (2) to corresponding BSs as a scheduling metric. The role of reference precoding ($\mathbf{P}_k$) is to keep the interference structure regardless of user scheduling and each user can estimate the quantity of the received interferences from other cells according to receive beamforming. Addition to the scheduling metric in (2), each user need to transmit its effective channel vector $\mathbf{u}_{[i,j]}{}^{*}\mathbf{H}_i^{[i,j]}\mathbf{P}_i$ from the correspondin BS, taking into account the receive beamforming, to the corresponding BS for downlink beamforming at the BS.

Figure 1 illustrates an example of MIMO IBC where $K = 3$, $M = 3$, $S = 2$, $L = 3$, and $N = 2$.

3) User Scheduling: Upon receiving N users' scheduling metrics in the serving cell, each BS selects S users having the smallest interference. Note again that we assume without loss of generality that the j-th users, $j = 1, \ldots, S$, in each cell have the smallest scheduling metrics and thus are selected.

4) Transmit Beamforming & Downlink Data Transmission: The transmit signal vector at the i-th BS for the j-th user in the i-th cell is given by $\mathbf{v}_{[i,j]}x^{[i,j]}$, where $x^{[i,j]}$ is the transmit symbol with power of $1/S$, and the transmit beamforming matrix for S users is given by $\mathbf{V}_i = \left[\mathbf{v}_{[1,i]}, \ldots, \mathbf{v}_{[S,i]}\right]$, where $\mathbf{v}_{[s,i]} \in \mathbb{C}^{S\times 1}$, $i \in \mathcal{K}$, $s \in \mathcal{S} \triangleq \{1, \ldots, S\}$. The transmit symbol vector of the i-th cell is given by $\mathbf{x}_i = \left[x^{[1,i]}, \ldots, x^{[S,i]}\right]^T$. Then, the received signal vector at the j-th user in the i-th cell can be written as:

$$\mathbf{y}^{[i,j]} = \underbrace{\mathbf{H}_i^{[i,j]}\mathbf{P}_i\mathbf{v}_{[i,j]}x^{[i,j]}}_{\text{desired signal}} + \underbrace{\sum_{s=1,s\neq j}^{S}\mathbf{H}_i^{[i,j]}\mathbf{P}_i\mathbf{v}_{[s,i]}x^{[s,i]}}_{\text{intra-cell interference}} + \underbrace{\sum_{k=1,k\neq i}^{K}\mathbf{H}_k^{[i,j]}\mathbf{P}_k\mathbf{V}_k\mathbf{x}_k}_{\text{inter-cell interference}} + \mathbf{z}^{[i,j]}, \tag{3}$$

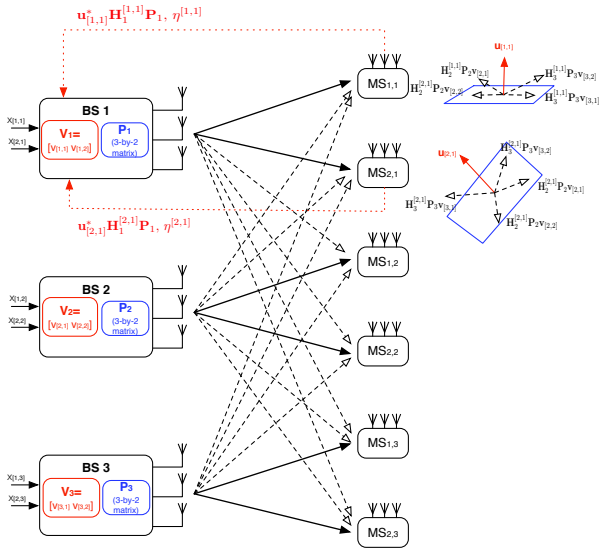

Fig. 1. MIMO IBC with $K = 3$, $M = 3$, $S = 2$, $L = 3$, and $N = 2$.

where $\mathbf{z}^{[i,j]} \in \mathbb{C}^{L\times 1}$ denotes the additive noise, each element of which is i.i.d. complex Gaussian with zero mean and the variance of SNR^{-1}. After receive beamforming at the j-th user in the i-th cell, the received signal vector can be rewitten as:

$$\tilde{y}^{[i,j]} = \mathbf{u}_{[i,j]}{}^{*}\mathbf{H}_i^{[i,j]}\mathbf{P}_i\mathbf{v}_{[i,j]}x^{[i,j]} + \mathbf{u}_{[i,j]}{}^{*}\mathbf{H}_i^{[i,j]}\mathbf{P}_i\sum_{s=1,s\neq j}^{S}\mathbf{v}_{[s,i]}x^{[s,i]} + \mathbf{u}_{[i,j]}{}^{*}\sum_{k=1,k\neq i}^{K}\mathbf{H}_k^{[i,j]}\mathbf{P}_k\mathbf{V}_k\mathbf{x}_k + \mathbf{u}_{[i,j]}{}^{*}\mathbf{z}^{[i,j]}, \tag{4}$$

The linear zero-forcing (ZF) beamformer can be applied at the BSs in order to cancel the intra-cell interference among the selected users' signals. Specifically, the transmit beamforming matrix of the i-th cell is designed by:

$$\mathbf{V}_i = \begin{bmatrix}\mathbf{u}_{[1,i]}{}^{*}\mathbf{H}_i^{[1,i]}\mathbf{P}_i \\ \mathbf{u}_{[2,i]}{}^{*}\mathbf{H}_i^{[2,i]}\mathbf{P}_i \\ \vdots \\ \mathbf{u}_{[S,i]}{}^{*}\mathbf{H}_i^{[S,i]}\mathbf{P}_i\end{bmatrix}^{-1} \cdot \begin{bmatrix}\sqrt{\gamma^{[i,1]}} & 0 & \cdots & 0 \\ 0 & \sqrt{\gamma^{[i,2]}} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \sqrt{\gamma^{[i,S]}}\end{bmatrix},$$

where $\sqrt{\gamma^{[i,j]}}$ denotes a normalization factor for satisfying the transmit power constraint. Then, the received signal is given as:

$$\tilde{y}^{[i,j]} = \sqrt{\gamma^{[i,j]}}x^{[i,j]} + \mathbf{u}_{[i,j]}{}^{*}\sum_{k=1,k\neq i}^{K}\mathbf{H}_k^{[i,j]}\mathbf{P}_k\mathbf{V}_k\mathbf{x}_k + \mathbf{u}_{[i,j]}{}^{*}\mathbf{z}^{[i,j]},$$

where the intra-cell interference from other scheduled users in the same cell is removed.

From (5), the achievable rate of the j-th user in the i-th cell is given by

$$R^{[i,j]} = \log_2\left(1 + \text{SINR}^{[i,j]}\right) = \log_2\left(1 + \frac{\gamma^{[i,j]}/S \cdot \text{SNR}}{1 + \tilde{I}^{[i,j]}}\right), \tag{5}$$

where $\tilde{I}^{[i,j]} = \sum_{k=1,k\neq i}^{K}\sum_{s=1}^{S}\left|\mathbf{u}_{[i,j]}{}^{*}\mathbf{H}_k^{[i,j]}\mathbf{P}_k\mathbf{v}_{[k,s]}\right|^2 \cdot \text{SNR}$.

IV. DoF Achievability

For given channel instance, from (5), each selected user can achieve the optimal DoF of 1 if and only if the interference $\tilde{I}^{[i,j]}$ remains

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constant for increasing SNR. Since $R^{[i,j]}$ can be bounded as

$$R^{[i,j]} \geq \log_2\left(1+\frac{\gamma^{[i,j]}/S\cdot \mathsf{SNR}}{1+\left\|\mathbf{v}_i^{(\max)}\right\|^2 I^{[i,j]}}\right), \quad (6)$$

where $\mathbf{v}_i^{(\max)}$ is defined by

$$\mathbf{v}_i^{(\max)} = \arg\max\left\{\left\|\mathbf{v}_{[i',j']}\right\|^2 : i'\in\mathcal{K}\setminus\{i\}, j'\in\mathcal{S}\right\}, \quad (7)$$

and $I^{[i,j]}$ is defined by

$$I^{[i,j]} \triangleq \sum_{k=1,k\neq i}^{K}\sum_{s=1}^{S}\left\|\mathbf{u}_{[i,j]}{}^*\mathbf{H}_k^{[i,j]}\mathbf{P}_k\mathbf{v}_{[k,s]}\right\|^2\cdot\mathsf{SNR}, \quad (8)$$

the optimal DoF can be achieved at each user if $I^{[i,j]}<\epsilon, \quad \forall j\in\mathcal{S}, i\in\mathcal{K}$, for some $0\leq\epsilon<\infty$.

A. Beamforming Weight Design

To maximize the achievable DoF, we aim to minimize the sum-interference $\sum_{i=1}^{K}\sum_{j=1}^{S} I^{[i,j]}$ through receive beamforming at the users. As in [16], [17], the following fruitful relation between the scheduling metrics and the sum-interference is used:

$$\sum_{i=1}^{K}\sum_{j=1}^{S} I^{[i,j]} = \sum_{i=1}^{K}\sum_{j=1}^{S}\eta^{[i,j]}\cdot\mathsf{SNR}. \quad (9)$$

This interestingly implies that the collection of distributed effort from the users to minimize $\eta^{[i,j]}$ can reduce the sum-interference. Therefore, each user finds the beamforming vector from

$$\mathbf{u}_{[i,j]} = \arg\min_{\mathbf{u}}\eta^{[i,j]} = \arg\min_{\mathbf{u}}\left\|\mathbf{G}^{[i,j]}\mathbf{u}\right\|^2, \quad (10)$$

where

$$\mathbf{G}^{[i,j]} \triangleq \left[\left(\mathbf{H}_1^{[i,j]}\mathbf{P}_1\right),\ldots,\left(\mathbf{H}_{i-1}^{[i,j]}\mathbf{P}_{i-1}\right),\left(\mathbf{H}_{i+1}^{[i,j]}\mathbf{P}_{i+1}\right),\ldots,\left(\mathbf{H}_K^{[i,j]}\mathbf{P}_K\right)\right]^* \in \mathbb{C}^{(K-1)S\times L}. \quad (11)$$

Let us denote the singular-value decomposition (SVD) of $\mathbf{G}^{[i,j]}$ as

$$\mathbf{G}^{[i,j]} = \mathbf{\Omega}^{[i,j]}\mathbf{\Sigma}^{[i,j]}\mathbf{V}^{[i,j]*}, \quad (12)$$

where $\mathbf{\Omega}^{[i,j]}\in\mathbb{C}^{(K-1)S\times L}$ and $\mathbf{V}^{[i,j]}\in\mathbb{C}^{L\times L}$ consist of L orthonormal columns, and $\mathbf{\Sigma}^{[i,j]} = \mathrm{diag}\left(\sigma_1^{[i,j]},\ldots,\sigma_L^{[i,j]}\right)$, where $\sigma_1^{[i,j]}\geq\cdots\geq\sigma_L^{[i,j]}$. Then, the optimal $\mathbf{u}^{[i,j]}$ is determined as

$$\mathbf{u}_{[i,j]} = \mathbf{v}_{[i,j]}^{(L)}, \quad (13)$$

where $\mathbf{v}_{[i,j]}^{(L)}$ is the L-th column of $\mathbf{V}^{[i,j]}$. With this choice the scheduling metric is simplified to

$$\eta^{[i,j]} = {\sigma_L^{[i,j]}}^2. \quad (14)$$

Since each column of $\mathbf{P}_k$ is isotropically and independently distributed, the effective interference channel matrix $\mathbf{G}^{[i,j]}$ is i.i.d. complex Gaussian with zero mean and unit variance.

To derive the achievable DoF, we start with the following lemmas.

Lemma 1: Suppose that the cumulative density function (CDF) of $\eta^{[i,j]}$ can be written without loss of generality by

$$F_\eta(x) = c_0x^\tau + o\left(x^\tau\right) \quad (15)$$

for $x>0$, where $\tau\triangleq(K-1)S-L+1$ and c_0 is a non-zero coefficient independent of x. Then the sum-interference remains constant with high probability for increasing SNR, that is,

$$\mathsf{P}\triangleq\lim_{\mathsf{SNR}\to\infty}\Pr\left\{\sum_{i=1}^{K}\sum_{j=1}^{S}I^{[i,j]}\leq\epsilon\right\}=1 \quad (16)$$

for any $0<\epsilon<\infty$, if

$$N=\omega\left(\mathsf{SNR}^\tau\right). \quad (17)$$

Proof: Using (9), P can be bounded by

$$\mathsf{P} = \lim_{\mathsf{SNR}\to\infty}\Pr\left\{\sum_{i=1}^{K}\sum_{j=1}^{S}\eta^{[i,j]}\cdot\mathsf{SNR}\leq\epsilon\right\} \quad (18)$$

$$\geq \lim_{\mathsf{SNR}\to\infty}\Pr\left\{\eta^{[i,j]}\leq\frac{\mathsf{SNR}^{-1}\epsilon}{KS^2},\forall i\in\mathcal{K},\forall j\in\mathcal{S}\right\} \quad (19)$$

Note that the selected users' $\eta^{[i,j]}$ are the minimum S values out of N i.i.d. random variables. If we denote a random variable with the same distribution of $\eta^{[i,j]}$ by η, (19) can be written by

$$\mathsf{P}\geq\lim_{\mathsf{SNR}\to\infty}\left[1-\sum_{i=1}^{S-1}\binom{N}{i}A^i(1-A)^{N-i}\right] \quad (20)$$

$$\geq\lim_{\mathsf{SNR}\to\infty}\left[1-\sum_{i=1}^{S-1}N^iA^i(1-A)^{-i}(1-A)^N\right], \quad (21)$$

where $A\triangleq F_\eta\left(\frac{\epsilon\mathsf{SNR}^{-1}}{KS^2}\right)$. From (15), we have

$$(1-A)^N=\left(1-c_0\left(\frac{\epsilon}{KS^2}\right)^\tau\cdot\mathsf{SNR}^{-\tau}+o\left(\mathsf{SNR}^{-\tau}\right)\right)^N. \quad (22)$$

Thus, $(1-A)^N$ tends to 0 (exponentially) if and only if N scales faster than SNR^τ. Now, inserting $N=\omega\left(\mathsf{SNR}^\tau\right)$ to (21) yields P tending to 1 for increasing SNR for given i, which proves the Lemma. ∎

Lemma 2 (Lemma 1 [17]): The CDF of $\eta^{[i,j]}$, denoted by $F_\eta(x)$, can be written as

$$F_\eta(x)=a_0x^{(K-1)S-L+1}+o\left(x^{(K-1)S-L+1}\right), \quad (23)$$

for $0\leq x<1$, where a_0 is a constant determined by K, S, and L.

Finally, the following theorem establishes the DoF achievability of the proposed OIA.

Theorem 1 (User scaling law: Downlink IBC): The proposed downlink OIA scheme with the scheduling metric (14) achieves

$$\mathrm{DoF}\geq KS \quad (24)$$

with high probability if

$$N=\omega\left(\mathsf{SNR}^{(K-1)S-L+1}\right). \quad (25)$$

Proof: If the sum-interference remains constant for increasing SNR with probability P, the achievable rate in (6) can be further bounded by

$$R^{[i,j]}\geq\left[\log_2\left(\mathsf{SNR}\right)+\log_2\left(1+\frac{\gamma^{[i,j]}/\left(S\left\|\mathbf{v}_i^{(\max)}\right\|^2\right)}{1/\left\|\mathbf{v}_i^{(\max)}\right\|^2+\epsilon}\right)\right]\cdot\mathsf{P},$$

for any $0\leq\epsilon<\infty$. Thus, the achievable DoF can be bounded by

$$\mathrm{DoF}\geq KS\cdot\mathsf{P}. \quad (26)$$

From Lemmas 1 and 2, it is immediate to show that P tends to 1, and hence KS DoF is achievable if $N=\omega\left(\mathsf{SNR}^{(K-1)S-L+1}\right)$, which proves the theorem. ∎

Now, to relate the obtained user scaling law to the interference decaying rate with respect to N for given SNR, we introduce the

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following lemma.

Lemma 3: Suppose that the CDF of $\eta^{[i,j]}$ can be written without loss of generality by (15). Then, the decaying rate of the interference received at a selected user with respect to N is given by

$$\chi \triangleq E\left\{\frac{1}{I^{[i,j]}}\right\} \leq O\left(N^{1/\tau}\right). \tag{27}$$

Proof: For given S, suppose the worse performance case where N users are divided into S subgroups with N/S users per each and where one user with the minimum $\eta^{[i,j]}$ is selected for each subgroup. Thus, $\eta^{[i,j]}$ is the minimum of N/S i.i.d. random variables. Then, the lemma can be proved following the footsteps of [22, Theorem 3]. Specifically, let us define α such that

$$\Pr\left\{\eta^{[i,j]} \leq \frac{1}{\alpha}\right\} = \frac{S}{N}. \tag{28}$$

From (15), we get

$$\Pr\left\{\eta^{[i,j]} \leq \frac{1}{\alpha}\right\} = c_0\alpha^{-\tau} + o\left(\alpha^{-\tau}\right). \tag{29}$$

From the equality between (28) and (29), we get $N^{-1} = O\left(\alpha^{-\tau}\right)$, and thus

$$\alpha = O\left(N^{1/\tau}\right). \tag{30}$$

In addition, since $1/\eta^{[i,j]}$ is the maximum out of N/S reversed scheduling metrics, it can be shown from (28) that

$$\Pr\left\{\frac{1}{\eta^{[i,j]}} \leq \alpha\right\} = \left(1 - \frac{1}{N/S}\right)^{N/S}. \tag{31}$$

Now, the Markov inequality yields

$$E\left\{\frac{1}{\eta^{[i,j]}}\right\} \geq \alpha \cdot \Pr\left\{\frac{1}{\eta^{[i,j]}} \geq \alpha\right\} \tag{32}$$

$$= \alpha \cdot \left(1 - \left(1 - \frac{1}{N/S}\right)^{N/S}\right) \tag{33}$$

$$= O\left(N^{1/\tau}\right), \tag{34}$$

where (34) follows from (30) and the fact that $\left(1 - \frac{1}{N/S}\right)^{N/S}$ converges to a constant for increasing N. ■

Theorem 2: If the user scaling law is given by $N = (\mathsf{SNR}^{\tau})$, then the interference decaying rate is given by

$$E\left\{\frac{1}{I^{[i,j]}}\right\} \leq O\left(N^{1/\tau}\right). \tag{35}$$

Proof: Since both the user scaling law and interference decaying rate are determined by the tail CDF of the scheduling metric, it is not difficult to prove the theorem using the proofs of Theorem 1 and Lemma 3. ■

Corollary 1: The interference decaying rate of the proposed OIA for the MIMO IBC is given by

$$E\left\{\frac{1}{I^{[i,j]}}\right\} \leq O\left(N^{\frac{1}{(K-1)S-L+1}}\right). \tag{36}$$

Proof: The proof is immediate from Theorems 1 and 2. ■

Remark 1: The user scaling law characterizes the trade-off between the asymptotic DoF and number of users, i.e., the more number of users, the faster DoF achievability. In addition, from Theorem 2, the user scaling law also provides the information on the interference decaying rate with respect to N for given SNR.

V. Simulation Results

In this section, the performance of the proposed OIA scheme is evaluated in comparison to the two existing schemes based on random beamforming at the BSs. First, the max-SNR scheme is considered as a base line scheme, in which the receiver beamforming as well as the user selection is performed only to maximize the gain of desired channels. Second, the random beamforming scheme with the minimum-leakage-of-interference (LIF) OIA is considered [13], [18], [21]. In the random beamforming scheme, no zero-forcing precoding is employed at the BSs, i.e., $\mathbf{V}_k = \mathbf{I}_S$, and hence intra-cell interference is not canceled at the users but only suppressed through the user scheduling and receiver beamforming. For more details, the readers are referred to [13], [18], [21].

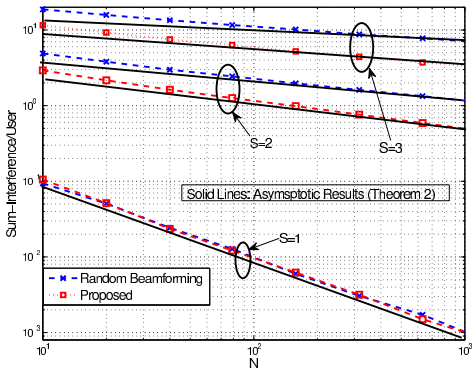

Fig. 2. Normalized sum-interference vs. N when $K = 3$, $M = 4$, $L = 2$.

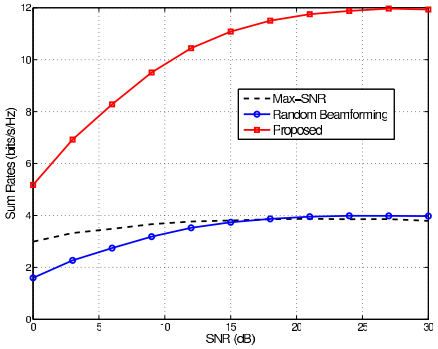

Fig. 3. Sum-rates vs. SNR when $K = 3$, $M = 4$, $L = 2$, and $N = 20$.

Fig. 2 shows the normalized sum-interference versus N when $K = 3$, $M = 4$, $L = 2$, and SNR=20dB for various S values. Since both the random beamforming and proposed schemes are based on the OIA frame work, interference decaying rates with respect to N follow Theorem 2. Since the user scaling law of the random beamforming scheme is given by $N = \omega\left(\mathsf{SNR}^{KS-L}\right)$ the interference decaying rate is given by $O\left(N^{-1/(KS-L)}\right)$, while it is $O\left(N^{-1/((K-1)S-L+1)}\right)$ for the proposed scheme. As a result, the proposed scheme exhibits equal or faster interference decaying rates than the random beamforming scheme, as shown in Fig. 2.

Fig. 3 illustrates the sum-rates versus SNR when $K = 3$, $M = 4$, $L = 2$, $S = 3$, and $N = 20$. Surprisingly, the rate of the random beamforming scheme is even lower than the max-SNR scheme especially in the low to mid SNR regime, because N is not large enough to suppress both the intra-cell and intercell interference. In such noise-limited case, the max-SNR sense is better than the min-LIF sense. On the other hand, the proposed scheme shows always higher sum-rates than the others, exploiting the benefit of completely canceled intra-cell interference.

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