On the Achievable Degrees-of-Freedom by Distributed Scheduling in (N, K)-User Interference Channels

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***Abstract*—We investigate achievable degrees-of-freedom (DoF) of an (N, K)-user interference channel where only K user (transmitter-receiver) pairs among N user pairs are allowed to simultaneously communicate in a dense network ($N \gg K$). Each node is assumed to have M antennas and to be randomly located. We propose a distributed scheduling protocol to achieve the maximum DoF (i.e., MK), which sequentially and opportunistically selects a user pair causing/receiving interference lower than a pre-determined threshold to/from already selected user pairs in each step. It is proven that the proposed protocol achieves the maximum DoF, MK, in the (N, K)-user interference channel with less stringent network size N, compared with the conventional centralized protocol which has been known as the best. With zero-forcing detector at receiver, we prove that it is sufficient that the network size N scales at least as $\omega\left(\mathrm{SNR}^{M^2K(K-1)}\right)$ to achieve the maximum number of DoF MK, where SNR denotes the received signal-to-noise ratio. We also investigate the required feedback overheads of the proposed protocol and show that it is quite small when the network is strongly interference-limited because only a small number of users are required to transmit their signaling. Our numerical results show that our proposed scheme controls interference more effectively than the centralized protocol.**

***Index Terms*—Interference channel, wireless ad hoc network, distributed scheduling, multi-user diversity, degrees-of-freedom.**

I. Introduction

DUE to increasing demands on multi-media services, wireless networks gradually become interference-limited and interference between communication links has been considered as a critical problem. Correspondingly, understanding fundamental characteristics of *interference channel* (IC) capacity becomes more important. For two-user IC, the capacity region was shown in the very strong and strong interference cases [1], [2] but it still remains unknown in the moderate interference region. In [3], Han and Kobayashi proposed an achievable scheme which has been considered as the best inner bound on two-user IC capacity. Recently, Etkin *et al*. found the rate region of two-user IC within one bit of capacity [4]. For more than two-user interference channels, the achievable degrees-of-freedom (DoF) has been widely used for capacity approximation due to difficulty of finding the exact capacity region [5]–[7]. It was shown that every user can achieve $\frac{1}{2}$ DoF in K-user IC through interference alignment [6]. However, it may not be easy to apply IA to ad hoc networks because IA requires global channel knowledge in general.

On the other hand, opportunistic scheduling using multi-user diversity has received much attention for better utilizing fading phenomenon in wireless networks. The opportunistic scheduling was originally proposed to maximize overall throughput in the uplink cellular network [8]. The basic idea of opportunistic scheduling is for a base station to select the user which has the best (on-peak) channel at a specific time. Hence, multi-user diversity gain comes from taking advantage of the characteristic of time-varying fading channels across different users. After the initial work in [8], there have been many works on opportunistic scheduling to obtain multi-user diversity in centralized networks [9]–[15]. In addition, opportunistic distributed scheduling has also been proposed in decentralized networks [16]–[21]. In decentralized networks, however, it is not easy to obtain channel state information (CSI) of other users since the infrastructure like base station does not exist. Accordingly, the design of a distributed opportunistic scheduling algorithm is considered very challenging and the existing studies provide only sub-optimal scheduling methods compared with centralized scheduling.

A. Related Works and Motivation

Recently, Zheng *et al.* proposed an opportunistic interference management (OIM) scheme which exploits multi-user diversity and investigated a capacity scaling law in an extended wireless ad hoc network [18]. In the OIM scheme, pre-determined transmitters broadcast pilot signals in a sequel and all nodes except for the transmitters compute SNRs from each transmitter to themselves. A node is selected as a receiver if SNR from only one transmitter among all transmitters is greater than a certain threshold and SNRs for remaining transmitters are below another threshold. The authors proved that an aggregate throughput scaling $\Theta\left(\frac{\log T(n)}{\sqrt{nT(n)}}\right)$ with communi-

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cation range of $T(n) = \Omega(\sqrt{\log n})$ is achievable through the scheme. In practice, however, a limited number of transmitter-receiver pairs are likely to be activated among the total number of nodes due to restrictions on available resources or scheduling complexity. This scenario is well modeled by an (N, K)-user interference channel embedded in a dense network where only K transmitter-receiver pairs among total N transmitter-receiver (i.e., user) pairs are allowed to communicate at a time. The resource allocation scenario for the downlink in a multi-cell network is one of practical examples [22], [23].

Recently, the achievable DoF and opportunistic gain of the (N, K)-user interference channel embedded in a dense network were investigated in [24], [25]. The authors showed that DoF of $d \in [0, MK]$ is achievable by using the centralized sub-group scheduling if the network-size scaling law satisfies a certain condition, where $M(> 1)$ indicates the number of antennas at each user. Specifically, for a single-input single-output (SISO) case, $d \in [0, K]$ is achievable if the network size scales like $N \in \omega(\mathrm{SNR}^{d\lceil d-1 \rceil})$ with power optimization and $N \in \omega(\mathrm{SNR}^{d(K-1)})$ without power optimization, respectively. In a multiple-input multiple-output (MIMO) case, a sufficient condition to achieve DoF of $d \in [0, MK]$ is that $N \in \omega(\mathrm{SNR}^{d\lceil d-1 \rceil})$ when power optimization is allowed. Without power optimization, the sufficient condition is given by $N \in \omega(\mathrm{SNR}^{d^2+d(K-2)M})$ if $2M - 1 \geq d$ and $N \in \omega(\mathrm{SNR}^{d(MK-1)})$ if $2M - 1 \leq d$, respectively.

However, it was assumed in [24], [25] that the whole network is divided into $\lfloor \frac{N}{K} \rfloor$ disjoint sub-groups consisting of K user pairs or MK antenna pairs. Each sub-group computes their achievable rate and reports it to the central unit. Then, the central unit selects the best sub-group achieving the maximum rate. Even though they well analyzed the achievable DoF and the required user scaling law for an arbitrary number of DoF (i.e., $d \in [0, MK]$), its achievable scheme is difficult to implement in infra-less networks such as ad hoc networks.

B. Summary of Contributions

In this paper, we propose a user scheduling method achieving the maximum DoF of the (N, K)-user interference channel where transmitters and receivers are equipped with multiple M antennas each. Contrary to the centralized approach in [25], the proposed scheduling operates in a *distributed* manner. In the proposed scheduling, the transmitter-receiver pair which causes/receives interference to/from previously selected transmitter-receiver pairs below a certain threshold level is sequentially selected among transmitter-receiver pairs. We show that the maximum DoF K for SISO case and MK for MIMO case are achievable by the proposed scheduling method under the network size scaling law $N = \omega\big(\mathrm{SNR}^{K(K-1)}\big)$ and $N = \omega\big(\mathrm{SNR}^{M^2K(K-1)}\big)$, respectively, when the codeword length is large enough. Especially, for achieving the maximum DoF MK in MIMO, the scaling law of the proposed scheduling is less stringent (i.e., lower) than the previous scaling law obtained from the centralized scheduling method in [25]. It is very interesting because the proposed scheduling method improves the theoretical scaling law even though it operates in a distributed manner. In addition, we investigate the required feedback overheads of the proposed protocol for validating practical feasibility. We find out that the feedback overheads per user become negligible in the high SNR regime where the network becomes strongly interference-limited because only a small number of users in the network are required to transmit their signaling data in the proposed protocol.

C. Organization

The rest of this paper is organized as follows. In Section II, we describe (N, K)-user interference channel model considered in this paper. In Section III, the distributed, opportunistic and sequential user scheduling protocol is proposed. The achievable DoF is mathematically analyzed in Section IV and feedback overheads of the proposed protocol are discussed in Section V. Numerical examples are presented in Section VI and conclusions are followed in Section VII.

D. Notations

Throughout the paper, we use the following notations. We denote the vector by lowercase bold letters and the matrix by uppercase bold letter. $\mathbf{A}^T, \mathbf{A}^H, \mathbf{A}^\dagger$ denote the transpose, conjugate transpose and pseudo-inverse of matrix $\mathbf{A}$, respectively. The Frobenius norm of a matrix A is expressed as $\|\mathbf{A}\|_F = \sqrt{\mathrm{Tr}(\mathbf{A}\mathbf{A}^H)}$, where $\mathrm{Tr}(\cdot)$ indicates the trace operation. We use the following notation if two functions $f(n)$ and $g(n)$ have the following relationship [26]:

- $f(n) = \omega(g(n))$ if $\lim_{n\to\infty} \frac{f(n)}{g(n)} = \infty$.
- $f(n) = \Omega(g(n))$ if $\lim_{n\to\infty} \frac{f(n)}{g(n)} > 0$.
- $f(n) = \Theta(g(n))$ if $\lim_{n\to\infty} \frac{f(n)}{g(n)} = c$, where $0 < c < \infty$.

II. System Model

For given positive integers N and K ($N \gg K$), we consider an (N, K)-user interference channel in an ad-hoc network where N user pairs (i.e., transmitter-receiver pairs) are randomly and densely ($N \to \infty$) distributed in 2-dimensional space within a finite area. The signal sent from one transmitter reaches all other transmitter-receiver pairs in the network. Each transmitter is assumed to communicate with only its designated receiver without help of relays. In the (N, K)-user interference channel, only K transmitters among N transmitter-receiver pairs are allowed to transmit independent data streams to their corresponding receivers simultaneously (i.e., *active* user pairs) at each time slot. Consequently, the selected K transmitter-receiver pairs construct the K-user interference channel and the remaining $N - K$ user pairs do not transmit (i.e., *inactive* user pairs). We assume that the wireless fading channels are time-invariant and each user is equipped with M antennas.

We define U as the set of indices of all user pairs in the network and S_k as the set of indices of the k user pairs selected until the k-th step, where $1 \leq k \leq K$. Hence, $|S_k| = k$ and $S_k \subset S_p$ for all $p \geq k$. Similarly, we define $S_k^c = U \backslash S_k$ as the set of indices of remaining user pairs after the k-th user pair is selected. In the proposed scheduling protocol, the user pairs are sequentially selected. After selecting K user pairs, K transmitters in the set S_K send their own data simultaneously and construct a K-user interference channel. Without loss of generality, we denote the indices of the K-selected user pairs

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as $1, 2, \ldots, K$ for mathematical simplicity. After K user pairs are selected at time t, the received signal at the selected j-th receiver is

$$\bar{\mathbf{y}}_j[t] = \sqrt{\gamma_{j,j}}\ \mathbf{G}_j[t]\mathbf{H}_{j,j}[t]\mathbf{x}_j[t] + \sum_{i=1, i\neq j}^{K} \sqrt{\gamma_{i,j}}\ \mathbf{G}_j[t]\mathbf{H}_{i,j}[t]\mathbf{x}_i[t] + \bar{\mathbf{z}}_j[t], \quad (1)$$

where $\sqrt{\gamma_{i,j}}$ (< 1) represents the pathloss between the active i-th transmitter and the j-th receiver $(i, j \in S_K)$. $\mathbf{H}_{i,j}[t] \in \mathbb{C}^{M\times M}$ indicates the fading channel matrix between the active i-th transmitter and the j-th receiver and each element of channel matrix is modeled by an independent and identically distributed (i.i.d.) complex Gaussian random variable with zero mean and unit variance. $\mathbf{G}_j[t] \in \mathbb{C}^{M\times M}$ is the zero-forcing (ZF) matrix of $\mathbf{H}_{j,j}[t]$ and each row vector of $\mathbf{G}_j[t]$ has unit norm. $\mathbf{z}_j[t] \in \mathbb{C}^{M\times 1}$ is an i.i.d. additive complex Gaussian noise vector with zero mean and unit variance at the j-th receiver. $\bar{\mathbf{z}}_j[t] \in \mathbb{C}^{M\times 1}$ denotes the ZF filtered output of the noise vector $\mathbf{z}_j[t]$ at the j-th receiver. $\bar{\mathbf{y}}_j[t] \in \mathbb{C}^{M\times 1}$ is the received signal at the j-th ZF receiver. $\mathbf{x}_i[t] \in \mathbb{C}^{M\times 1}$ represents the transmitted signal of the i-th transmitter. Each transmitter i satisfies the average power constraint $\mathbb{E}[\mathbf{x}_i\mathbf{x}_i^H] = \mathrm{SNR}$, where SNR denotes the signal-to-noise ratio. For convenience, we will omit the time index t in the following sections.

III. Distributed, Opportunistic and Sequential User Scheduling

In this section, we propose a distributed user pair scheduling protocol to achieve maximum DoF MK in the (N, K)-user interference channel. The proposed protocol is based on time division duplex (TDD) so that the channel reciprocity is assumed. In the proposed protocol, K active user pairs are *opportunistically* and *sequentially* selected in a *distributed* manner by utilizing pilot signal (or reference signal) to estimate channel at each user pair. We assume that all user pairs in the network are synchronized with the help of some distributed synchronization protocols or GPS [33], [34] [1]. We assume that time duration for exchanging the pilot signal is short enough to be negligible compared to data-packet transmission time.

The first transmitter-receiver pair is randomly selected among N user pairs and the index of this randomly selected user pair belongs to S_1. This can be implemented by a random access scheme adopting random backoff time at each transmitter [28]–[30]. We define the caused interference from an arbitrary transmitter s in S_k^c to the k selected receivers in the k-th selection step (i.e., after k user pairs are selected) as

$$I_k^{s,c} = \sum_{j=1}^{k} \gamma_{s,j} \|\mathbf{G}_j\mathbf{H}_{s,j}\|_F^2 \quad (2)$$

where $\mathbf{G}_j = [\mathbf{g}_j^{(1)}, \cdots, \mathbf{g}_j^{(M)}]^H$ and $\mathbf{g}_j^{(n)} \in \mathbb{C}^{M\times 1}$ is the unit-norm n-th column vector of the ZF matrix $\mathbf{H}_{j,j}^{\dagger}$ at the receiver $s \in S_k^c$ for each $k \in \{1, \cdots, K-1\}$. Similarly, we also define the received interference from the k selected transmitters to an arbitrary receiver s in S_k^c in the k-th selection step (i.e., after k user pairs are selected) as

$$I_k^{s,r} = \sum_{i=1}^{k} \gamma_{i,s} \|\mathbf{G}_s\mathbf{H}_{i,s}\|_F^2. \quad (3)$$

Note that the caused and received interference powers are calculated in the second user pair selection step for the first time since the first user pair is assumed to be randomly selected without any consideration of interference. Hence, (2) and (3) can be used as a metric for the $(k+1)$-th user pair selection in our proposed scheduling protocol, where $1 \le k \le K-1$.

At each user pair selection step in the proposed protocol, all candidate user pairs in S_k^c compute (2) and (3). Then, the user pair whose both transmitter and receiver simultaneously satisfy a specific threshold condition is selected and added to the set S_k. The user selection procedure is described in detail:

- *Step 1*: A randomly selected transmitter of the first user pair sends pilot signal to its designated receiver and the receiver sends back the pilot signal. The corresponding receiver additionally broadcasts the ZF matrix $\mathbf{G_1}$ of its own channel.
- *Step 2*: Each transmitter in S_1^c calculates its causing interference $I_1^{s,c}$ to the receiver of the first user pair by overhearing the broadcast ZF matrix and reference signal (in SISO case, through only reference signal) in Step 1 based on channel reciprocity. Similarly, each receiver in S_1^c can also calculate the received interference $I_1^{s,r}$ from the first transmitter by overhearing the broadcasted reference signal in Step 1 and by applying ZF detector corresponding to its desired channel.
- *Step 3*: Each receiver of user pair in S_1^c examines whether its received interference power is lower than a predefined threshold ϵ_1. In other words, each receiver checks whether the threshold condition is satisfied or not. If a receiver satisfies the threshold condition, then it sets a backoff time proportional to the amount of its received interference from the selected transmitter [28]–[30]. The waiting time due to the random backoff becomes smaller when the received interference is weaker. If we set a maximum allowable waiting time as $T_{\max}$, the random backoff time of a receiver with its received interference $I_1^{s,r}$ is determined by $T_{\max} \cdot \frac{I_1^{s,r}}{\epsilon_1}$, where ϵ_1 is the threshold of received interference. Since only receivers satisfying the threshold condition (i.e., $I_1^{s,r} \le \epsilon_1$) set the backoff time for sending feedback information, $\frac{I_1^{s,r}}{\epsilon_1}$ is less than or equal to 1. The maximum backoff time is restricted by maximum allowable waiting time $T_{\max}$. Collisions occur with high probability due to propagation delay and non-zero packet length if $T_{\max}$ is small, but sufficiently large $T_{\max}$ can prevent collisions. However, excessive large $T_{\max}$ results in long waiting time so that the tradeoff between collision probability and waiting time determines the optimal value of $T_{\max}$ as a controllable scaling factor. Assuming optimized $T_{\max}$, we can guarantee a negligible collision probability by the random backoff algorithm

[1] If we denote the imposed signaling overheads for synchronization and data transmission time as T_s and T, respectively, the DoF is written by $\frac{T}{T_s+T}MK$. However, the signaling overheads by distributed synchronization protocols can be negligible in terms of DoF if the data transmission time is long enough. If GPS is adopted, signaling overheads for time-synchronization is effectively avoided.

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because the probability that more than two users have the same received interference power from the selected users is almost surely zero. In our dense network model where the transmitted signal from one transmitter reaches all the other transmitter-receiver pairs, the propagation delay is not large. Furthermore, the packet length of the indicating signal is small since it only indicates whether the threshold condition is satisfied or not. Fortunately, these characteristics allow relatively small $T_{\max}$ for a negligible collision probability. Although even small $T_{\max}$ causes latency by the random backoff algorithm, the DoF loss due to the latency can be marginal; if we set the resultant latency and data transmission time as $T_b(T_{\max}, \epsilon_k, K, M)$ and T, respectively, the achievable DoF is given by $\frac{T}{T_b(T_{\max},\epsilon_k,K,M)+T} \cdot MK$. Total used backoff time can be modeled as a function of ϵ_k, maximum allowable waiting time $T_{\max}$, the number of selected users (K), and the number of antennas (M), since the backoff time is determined as $T_{\max} \cdot \frac{I_k^{s,r}}{\epsilon_k}$. The DoF loss caused by the random backoff algorithm is reflected by the multiplier $\frac{T}{T_b(T_{\max},\epsilon_k,K,M)+T}$. However, the resultant latency is negligible in the achievable DoF analysis because data transmission time is in general assumed to be long enough [31], [32]. Similarly, each transmitter in S_1^c also examines whether its causing interference to the receiver of the first user pair $I_1^{s,c}$ is lower than the threshold ϵ_1.

- *Step 4*: According to the backoff time in Step 3, the receiver with the minimum backoff time sends an indicating signal bearing the information whether the receiver satisfies the threshold condition or not back to its transmitter. If a transmitter receives the indicating signal from its receiver and it satisfies the threshold condition, it immediately sends back a probing signal to the receiver to notify that the user pair is selected as the second user pair and its index belongs to S_2. Then, the remaining candidate receivers which satisfy the threshold condition in S_1^c stop the waiting process for sending the indicating signal. Note that the second user pair satisfies the threshold condition so that both the caused and the received interference are at most ϵ_1. If the transmitter which receives the indicating signal from its receiver does not satisfy the threshold condition, then it does not send any signal (i.e., *being silent*) and the receiver with the second minimum backoff time sends the indicating signal to its transmitter. This process is repeated until both transmitter and receiver satisfy the threshold condition. If there is no selected user pair, an outage is declared, all nodes defer transmission until the next transmission time, and the protocol is reset. However, if N is sufficiently large, a user pair is surely selected in each selection step, which will be shown in the next section.
- *Step 5*: The receiver of the selected second user pair in Step 4 broadcasts a reference signal and its ZF matrix $\mathbf{G_2}$ of its own channel.
- *Step 6*: Similarly to Step 2, each transmitter and receiver in S_2^c calculates $I_2^{s,c}$ and $I_2^{s,r}$, respectively.
- *Step 7*: Through the same feedback operation of the indicating signal as Step 4, the third user pair is selected and then its index belongs to S_3.
- *Step 8*: The same user selection processes are repeated until K user pairs are selected. Then, the K user pairs transmit their data packet simultaneously.

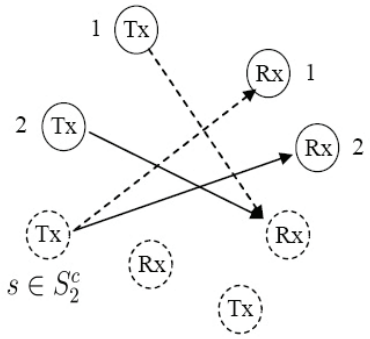

Fig. 1. Caused and received interference at arbitrary node pair s to select the third user pair among S_2^c. The firstly and secondly selected user pairs are denoted by 1 and 2, respectively. The interference denoted by dotted arrow is pre-calculated by overhearing the probing signal in the previous user pair (denoted by 1) selection. Therefore, the only required interference values to calculate $I_2^{s,c}$ and $I_2^{s,r}$ are the caused interference to Rx 2 (Tx $s \rightarrow$ Rx 2) and the received interference from Tx 2 (Tx 2 $\rightarrow$ Rx s).

Note that this user selection is *opportunistic*, *sequential* and *distributed*[2]. It is noteworthy that, since each user pair is selected sequentially, the pre-calculated values of the interference $I_{k-1}^{s,c}$ and $I_{k-1}^{s,r}$ in the selection of the k-th user pair can be reused to calculate $I_k^{s,c}$ and $I_k^{s,r}$ in the selection of the $k+1$-th user pair. Therefore, the received interference power from the k-th selected user pair and the causing interference power to the k-th selected user pair are only required to calculate $I_k^{s,c}$ and $I_k^{s,r}$. For better understanding, see Fig. 1 as an example. In Fig. 1, the dotted arrow line denotes the interference which was pre-calculated in the previous user selection steps and the bold dotted arrow line indicates the interference required to be calculated in the present user selection step.

IV. Performance Analysis

In this section, we analyze the achievable DoF of the proposed user scheduling protocol with ZF detector at receivers. It is proven that the required number of user pairs in a network is sufficient to be scaled as $\omega\left(\mathrm{SNR}^{M^2K(K-1)}\right)$ to achieve the maximum number of DoF MK, which is less stringent compared to the user scaling law in [25].

A. Achievable Degrees-of-Freedom

If K user pairs are selected by the proposed protocol in Section III and ZF detector is used at receivers, then the achievable rate at the j-th receiver for the n-th data stream

[2]Our proposed protocol relaxes some ideal assumptions of centralized protocols like global CSI and can be implemented by a random access protocol like RTS/CTS-based CSMA/CA without degrading DoF performance if the data packet length is sufficiently long. In this context, our proposed protocol is claimed as a distributed protocol.

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of the j-th transmitter is given as

$$R_j^{(n)} = \log\Big(1 + \frac{\gamma_{j,j}|\mathbf{g}_j^{(n)H}\mathbf{h}_{j,j}^{(n)}|^2\mathrm{SNR}}{1+\sum_{i=1,i\neq j}^{K}\sum_{m=1}^{M}\gamma_{i,j}|\mathbf{g}_j^{(n)H}\mathbf{h}_{i,j}^{(m)}|^2\mathrm{SNR}}\Big), \tag{4}$$

where $\mathbf{g}_j^{(n)} \in \mathbb{C}^{M\times 1}$ is the n-th column vector of the ZF matrix $\mathbf{H}_{j,j}^{\dagger}$ at the j-th receiver and $\mathbf{h}_{i,j}^{(m)}$ is the m-th column vector of the channel matrix between the i-th transmitter and the j-th receiver. From (4), the total achievable sum rate of K user pairs can be written as

$$\sum_{j=1}^{K} R_j(\mathrm{SNR}) = \sum_{j=1}^{K}\sum_{n=1}^{M}\log\Big(1+\frac{\gamma_{j,j}|\mathbf{g}_j^{(n)H}\mathbf{h}_{j,j}^{(n)}|^2\mathrm{SNR}}{1+\sum_{i=1,i\neq j}^{K}\sum_{m=1}^{M}\gamma_{i,j}|\mathbf{g}_j^{(n)H}\mathbf{h}_{i,j}^{(m)}|^2\mathrm{SNR}}\Big). \tag{5}$$

The achievable DoF by the proposed protocol in the (N,K)-user interference channel where M multiple antennas are equipped at each node and zero-forcing detector is adopted for receivers is given by

$$\sum_{j=1}^{N} d_j = \lim_{\mathrm{SNR}\to\infty}\frac{\sum_{j=1}^{K} R_j(\mathrm{SNR})}{\log(\mathrm{SNR})}, \tag{6}$$

where d_j is achievable DoF at the j-th user pair. We also define the caused and the received interference from the $s(\in S_k^C)$-th user pair without consideration of the pathloss term in (2) and (3), respectively, as

$$J_k^{s,c} = \sum_{j=1}^{k}\|\mathbf{G}_j\mathbf{H}_{s,j}\|_F^2 \quad \text{and} \tag{7}$$

$$J_k^{s,r} = \sum_{i=1}^{k}\|\mathbf{G}_s\mathbf{H}_{i,s}\|_F^2 \quad \text{for } 1\le k\le K-1. \tag{8}$$

Note that each projection of $\mathbf{h}_{s,i}^{(m)}$ on $\mathbf{g}_i^{(n)H}$ is a complex Gaussian vector with zero-mean and unit-variance since all M-dimensional channel vectors $\mathbf{h}_{s,i}^{(m)}$ of $\mathbf{H}_{s,i}$ are isotropically distributed and each ZF unit-norm vector $\mathbf{g}_i^{(n)H}$ of $\mathbf{G}_i$ is independent of $\mathbf{h}_{s,i}^{(m)}$ for all $m\in\{1,2,\cdots,M\}$. Similarly, $\mathbf{G}_s\mathbf{H}_{i,s}$ has also complex Gaussian elements with zero-mean and unit-variance. Therefore, $J_k^{s,c}$ and $J_k^{s,r}$ have Chi-square distributions with $2kM^2$ degrees of freedom for each $k\in\{1,2,\cdots,K-1\}$. If we represent $J_k^{s,c}$ and $J_k^{s,r}$ as a unified random variable J_k^s for simplification, the cumulative distribution function (CDF) of J_k^s is given by

$$F_{J_k^s}(l_k) = \frac{\gamma\left(kM^2, l_k/2\right)}{\Gamma\left(kM^2\right)}, \tag{9}$$

where $\Gamma(z)=\int_0^\infty t^{z-1}e^{-t}dt$ is the Gamma function and $\gamma(z,x)=\int_0^x t^{z-1}e^{-t}dt$ is the lower complete Gamma function.

Lemma 1. *When J_k^s has chi-square distribution with $2a$ degrees of freedom, the CDF of J_k^s is given by $F_{J_k^s}(l_k) = \frac{\gamma(a,l_k/2)}{\Gamma(a)}$. For any $0\le l_k<2$, the CDF $F_{J_k^s}$ of J_k^s is lower- and upper-bounded by*

$$C_k^1(l_k)^a \le F_{J_k^s}(l_k) \le C_k^2(l_k)^a, \tag{10}$$

where

$$C_k^1 = \frac{2^{-a}}{a\Gamma(a)}e^{-\frac{l_k}{2}},\ C_k^2 = \frac{2^{-a}}{a\Gamma(a)}\left(1+\frac{\frac{l_k}{2}}{a+1-\frac{l_k}{2}}\right),$$

and $\Gamma(z)=\int_0^\infty t^{z-1}e^{-t}dt$ is the Gamma function.

Proof: The proof of this lemma is placed in [27]. ■

Lemma 2. *In the $(k+1)$-th user selection step of our proposed scheduling protocol, the probability at least one transmitter-receiver pair satisfies the threshold condition ϵ_k among $N-k$ candidate transmitter-receiver pairs (i.e., link establishment probability) is given as*

$$1-\left\{1-\left(F_{J_k^s}\left(\epsilon_k SNR^{-1}\right)\right)^2\right\}^{N-k}, \tag{11}$$

where $F_{J_k^s}(\cdot)$ is CDF of J_k^s which has Chi-square distribution with $2kM^2$ degrees of freedom and $1\le k\le K-1$.

Proof: See Appendix A. ■

Theorem 1. *K user pairs are assumed to be selected by the proposed opportunistic and sequential user scheduling protocol in (N,K)-user interference channel where M multiple antennas are equipped at each node and zero-forcing detector is adopted for receivers. Then,*

$$\sum_{j=1}^{N} d_j = MK$$

is achievable if $N=\omega\left(\mathrm{SNR}^{M^2K(K-1)}\right)$.

Proof: See Appendix B. ■

Corollary 1. *In the SISO case, K user pairs are assumed to be selected by the proposed user selection protocol in the (N,K)-user interference channel. Then,*

$$\sum_{j=1}^{N} d_j = K$$

is achievable if $N=\omega\left(\mathrm{SNR}^{K(K-1)}\right)$.

Remark 1. *For the MIMO case, the proposed scheduling protocol achieves the maximum MK DoF with the less stringent user scaling compared to [25] even though user pairs are selected in a distributed manner in the proposed protocol.*

Remark 2. *For the SISO case, the resulting user scaling law of the proposed distributed scheduling protocol is the same as that of [25] which is based on centralized scheduling.*

V. Feedback Overheads and Technical Issues for Practical Implementation

A. Feedback Overheads

In the previous section, we analyzed the achievable DoF of the proposed protocol without consideration of signaling

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overheads (i.e., feedback of the indicating signal). If the codeword length of user data is long enough compared to the signaling message length [31], [32], the signaling overheads can be negligible in DoF analysis. In this subsection, to understand how relatively long should the codeword length be compared to the feedback overheads, we investigate the total amount of feedback overheads of the proposed protocol and quantify the burden of feedback overheads on each user.

Theorem 2. *When the entire K user pairs are selected via the proposed scheduling protocol adopting a finite threshold value ϵ_k independent of* SNR *for $1 \le k \le K-1$, the worst case feedback overheads are upper-bounded as*

$$\sum_{k=1}^{K-1} F_k \le \sum_{k=1}^{K-1} \frac{(N-k)\cdot 2^{-kM^2}}{kM^2\Gamma(kM^2)}\left(1+\frac{\tilde{l}_k/2}{kM^2+1-\tilde{l}_k/2}\right)(\tilde{l}_k)^{kM^2} \tag{12}$$

and the scaling of feedback in the high SNR is given by

$$\sum_{k=1}^{K-1} F_k \le \frac{N}{\mathrm{SNR}^{M^2}}, \tag{13}$$

where $\tilde{l}_k = \tilde{\epsilon}_k \mathrm{SNR}^{-1}$, $\tilde{\epsilon}_k = \epsilon_k/\gamma_{\min}$, and $\gamma_{\min} = \min_{1\le i\le k, s\in S_k^c} \gamma_{i,s}$.

Proof: If the receivers satisfy the threshold condition in each user pair selection step, they feed the indicating signal back to their transmitters by a backoff time based random access protocol (recall the feedback operation in Section III). In the $(k+1)$-th user pair selection step, the average number of receivers satisfying the threshold condition is

$$F_k = (N-k)\cdot \Pr\left[I_k^{s,r} \le \epsilon_k \mathrm{SNR}^{-1}\right] \tag{14}$$

$$= (N-k)\cdot \Pr\left[\sum_{i=1}^{k} \gamma_{i,s}\|\mathbf{G}_s\mathbf{H}_{i,s}\|_F^2 \le \epsilon_k \mathrm{SNR}^{-1}\right]. \tag{15}$$

By substituting the different pathloss terms $\{\gamma_{i,s}; i = 1, \cdots, k\}$ with the minimum one, (15) is upper-bounded as

$$F_k \le (N-k)\cdot \Pr\left[\gamma_{\min} J_k^{s,r} \le \epsilon_k \mathrm{SNR}^{-1}\right], \tag{16}$$

where

$$\gamma_{\min} = \min_{1\le i\le k, s\in S_k^c} \gamma_{i,s}. \tag{17}$$

Since $J_k^{s,r}$ has the chi-square distribution with $2kM^2$ degrees-of-freedom, (16) can be re-written by using CDF of J_k^s as

$$F_k \le (N-k)\, F_{J_k^s}(\tilde{l}_k), \tag{18}$$

where $\tilde{l}_k = \tilde{\epsilon}_k \mathrm{SNR}^{-1}$ and $\tilde{\epsilon}_k = \epsilon_k/\gamma_{\min}$. By using Lemma 1, (18) can be upper-bounded by

$$F_k \le (N-k)\frac{2^{-kM^2}}{kM^2\Gamma(kM^2)}\left(1+\frac{\tilde{l}_k/2}{kM^2+1-\tilde{l}_k/2}\right)(\tilde{l}_k)^{kM^2}. \tag{19}$$

In the proposed protocol, the receiver with the weakest interference firstly sends the indicating signal through the backoff time based random access protocol if it passed the threshold condition. Therefore, the worst case in terms of feedback occurrence corresponds to when the user pair with the receiver suffering from the strongest interference is selected after the sequential opportunistic feedback. The total feedback overheads (i.e., the number of feedback occurrences) in the network, $\sum_{k=1}^{K-1} F_k$, are upper bounded by

$$\sum_{k=1}^{K-1} F_k \le \sum_{k=1}^{K-1} \frac{(N-k)\cdot 2^{-kM^2}}{kM^2\Gamma(kM^2)}\left(1+\frac{\tilde{l}_k/2}{kM^2+1-\tilde{l}_k/2}\right)(\tilde{l}_k)^{kM^2}. \tag{20}$$

When SNR is sufficiently large, the term $1+\frac{\tilde{l}_k/2}{kM^2+1-\tilde{l}_k/2}$ in the right hand side goes to 1. Neglecting finite values related to k and ϵ_k in the high SNR, (20) is simplified by considering dominant scaling related to SNR as

$$\sum_{k=1}^{K-1} F_k \le \sum_{k=1}^{K-1} \frac{(N-k)\cdot 2^{-kM^2}}{kM^2\Gamma(kM^2)}\left(1+\frac{\tilde{l}_k/2}{kM^2+1-\tilde{l}_k/2}\right)(\tilde{l}_k)^{kM^2} \tag{21}$$

$$\simeq \sum_{k=1}^{K-1} \frac{N}{\mathrm{SNR}^{kM^2}} = \frac{N}{\mathrm{SNR}^{M^2}} \frac{1-\frac{1}{\mathrm{SNR}^{(K-1)M^2}}}{1-\frac{1}{\mathrm{SNR}^{M^2}}} \tag{22}$$

$$\simeq \frac{N}{\mathrm{SNR}^{M^2}}. \tag{23}$$

■

Note that (20) shows *the number of feedback occurrences* in the proposed protocol for *the worst case*. If we assume that each feedback of indicating signal consumes one time slot, the number of feedback occurrences can be interpreted as the total number of consumed time slots for feedback. We assume that the codeword length of user data is much longer than the total consumed time for feedback.

Note also that the scaled amount of total feedback overheads in the proposed protocol in the high SNR region where the network becomes strongly interference-limited is given by (23) for the worst case scenario. However, the typical amount of total feedback overheads might be much less than (23) since all remaining receivers do not send the indicating signal in the sequential feedback mechanism once a transmitter-receiver pair is selected. In the optimistic scenario, the total feedback overheads are only $K-1$ time slots since only 1 time slot is required in each user selection step. For a given finite SNR value, the feedback overheads are determined by computing the RHS of (20).

The feedback overheads certainly increase as K increases. For a given link establishment probability, in the process of adding a user pair, there exists an optimal threshold ϵ_k in terms of feedback overheads and the feedback overheads tend to linearly increase with K for the optimized ϵ_k. However, as shown in (20), the relationship among K, N, ϵ_k, and feedback overheads is very complicated. Thus, in order to obtain an insight about the relationship between K and the feedback overheads, we set ϵ_k to be $\gamma_{\min}$, which is independent of k and user index i. In this setting, the link outage probability is small enough because we use the minimum pathloss instead of the actual pathloss for each user, but the resultant feedback overheads are quite larger than the actual feedback overheads. When ϵ_k is $\gamma_{\min}$, the worst case feedback overheads are

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simplified as

$$\sum_{k=1}^{K-1} \frac{(N-k)\cdot 2^{-kM^2}}{kM^2\Gamma(kM^2)} \left(1+\frac{\frac{1}{2\mathrm{SNR}}}{kM^2+1-\frac{1}{2\mathrm{SNR}}}\right) \mathrm{SNR}^{-kM^2}. \tag{24}$$

The overhead burden per user can be quantified by dividing the total feedback overheads in the network by the required number of users for achieving the maximum DoF MK. In the previous section, the required user scaling law of the proposed scheme for DoF MK is obtained by $N = \omega(\mathrm{SNR}^{M^2K(K-1)})$. When $N = \omega(\mathrm{SNR}^a)$, any $N = \lceil \mathrm{SNR}^{a+\epsilon} \rceil$ for $\epsilon > 0$ is the smallest required number of users where $\epsilon \approx 0$ but $\epsilon \neq 0$. Therefore, for arbitrarily small ϵ (≈ 0), the required number of users in the proposed protocol for DoF MK is given by $N \approx \lceil \mathrm{SNR}^{M^2K(K-1)} \rceil$ in the high SNR region. From (23), the worst case feedback overheads per user in the high SNR region is given by $\rho_1 = \frac{\lceil \mathrm{SNR}^{M^2(K^2-K-1)} \rceil}{\lceil \mathrm{SNR}^{M^2K(K-1)} \rceil} \approx \mathrm{SNR}^{-M^2}$. In the low and mid SNR region, constant terms related k and ϵ_k cannot be neglected in calculation of the worst case overheads so that the worst case feedback overheads per user are defined by the ratio ρ_2 between (20) and $N = \lceil \mathrm{SNR}^{M^2K(K-1)} \rceil$. The worst case feedback overheads per user ρ_1 and ρ_2 will be shown according to SNR and M in Section VI.

B. Technical Issues for Practical Implementation

In this subsection, we discuss technical issues for practical implementation of the proposed protocol. We also enumerate all the necessary information exchanges required in the proposed protocol.

- *Channel State Information (CSI)*: CSI is used to calculate $I_k^{s,c}$ and $I_k^{s,r}$ at each transmitter and receiver. The CSI in the proposed protocol is assumed to be obtained through exchange of probing signals between the selected transmitter and its receiver. Note that any information exchange is not required for calculating $I_k^{s,r}$ because the channel is estimated by only observing the probing signals. On the other hands, calculation of $I_k^{s,c}$ requires the zero forcing matrices of the selected users in the previous selection steps. If the zero-forcing matrix is assumed to be quantized by b bits, then total $(K-1)b$ bits need to be exchanged for calculation of $I_k^{s,c}$ in the proposed protocol. This signaling overheads are relatively small if the codeword length is sufficiently large.
- *Time synchronization*: The operation of the proposed protocol primarily requires CSI by probing signal exchanges, the zero-forcing matrix, and indicating signals of 1 bit for status report. Time synchronization plays a key role in acquisition of the information and distributed coordination among users. For time synchronization, distributed time synchronization algorithms for wireless networks [33], [34] can be considered. Furthermore, GPS signals can also be used for time synchronization among users.
- *Activation of the selected users*: In the proposed protocol, all user pairs in the network can know which user is selected in each user selection step by observing the corresponding transmitter's probing signal. Note that we assumed the network where all users can overhear one another. When no user is selected at the $(k+1)$-th user pair selection step for $k \in \{1 \cdots K-1\}$, no probing signal is received at all user pairs within a time duration $T_{\max}+T_d$ where T_d indicates the maximum propagation delay in the network. Then, all user pairs perceive that only k user pairs (not K user pairs) are selected in this case and only k selected users send their messages simultaneously and remaining $N-k$ users become silent. Therefore, any information exchange is not necessary for activation of the selected users if the user pairs are time-synchronized.

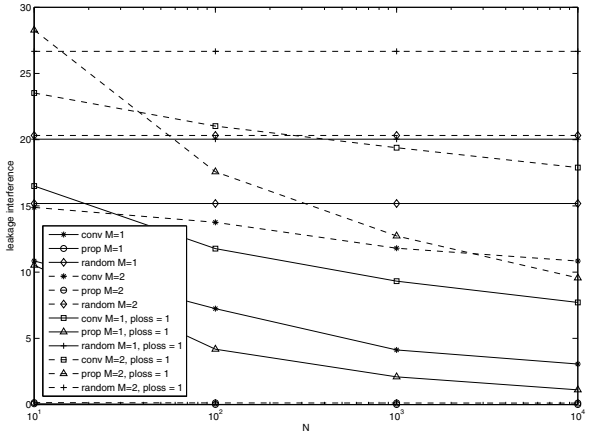

Fig. 2. Average leakage interference per antenna of the proposed scheduling protocol is compared with the conventional centralized scheme [25] for varying network size N. For the case that pathloss is assumed to be 1, the average leakage interference is normalized by 10^2.

VI. Numerical Results

In this section, we illustrate the analytical results in the previous sections. The area of the network we consider is a two dimensional square of which one side is defined as $[0, 500]$. All transmitter-receiver pairs are randomly distributed in the square. The pathloss is assumed to depend on only distance between transmitter and receiver and its pathloss exponent is equal to 2 (i.e., $d_{a,b}^{-2}$) where $d_{a,b}$ is the distance between transmitter a and receiver b. Transmit power of each transmitter SNR is set to be 30dB and the selected number of user pairs, K, is assumed to be 3, unless otherwise stated. The numbers of antennas at each node are 1 for SISO and 2 for MIMO, respectively.

Fig. 2 shows the average leakage interference per antenna with and without large-scale fading. The average leakage interference per antenna of the proposed scheme, the conventional centralized scheme [25], and a random scheduling scheme are compared according to the total number of user pairs in the network. In the random scheduling scheme, user pairs are randomly selected. The proposed scheme yields much lower interference than the centralized scheduling scheme as the network size N increases. This result implicates that the proposed scheme for both SISO and MIMO more effectively controls interference by exploiting multi-user diversity compared to the centralized scheme in the high SNR regime.

Fig. 3 compares the proposed scheduling with the centralized scheme [25] in terms of the required number of

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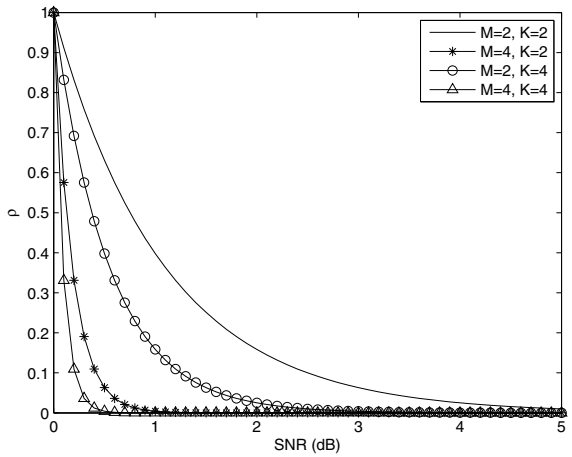

Fig. 3. The ratio between the required scaled number of users of the centralized scheme [25] and that of the proposed scheduling protocol versus SNR for various M and K.

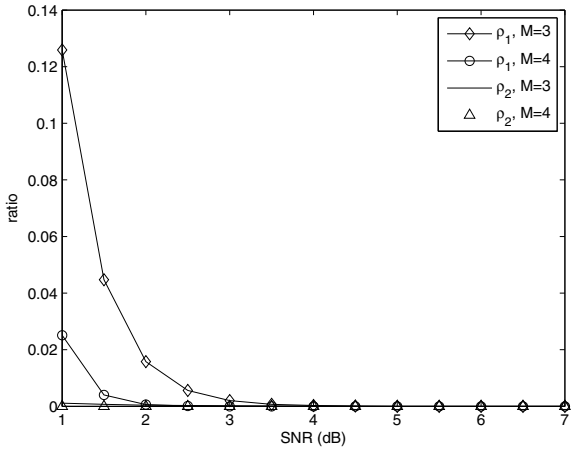

Fig. 4. Two ratios ρ_1 and ρ_2 (i.e., feedback overheads per user at the worst case) are shown according to SNR for various M when $K = 3$.

users (i.e., the network size) for achieving the maximum number of DoF MK. While the required user scaling of the centralized scheme for the maximum number of DoF MK is $N = \omega(\mathrm{SNR}^{M^2K^2+MK(K-2)M})$ for $2M-1 \geq MK$ and $N = \omega(\mathrm{SNR}^{MK(MK-1)})$ for $2M-1 \leq MK$, respectively, the required user scaling law of the proposed scheme is $N = \omega(\mathrm{SNR}^{M^2K(K-1)})$. For the purpose of scaling law comparison, we focus on the ratio ρ between the smallest numbers of users to achieve the maximum number of DoF MK in the centralized scheme and the proposed scheduling. Note that the *ratio* is used for meaningful comparison because our scaling law looks like 0 compared to that of the centralized protocol proposed in [25]. From the obtained scaling of the required number of users, $\rho = \frac{\lceil \mathrm{SNR}^{M^2K(K-1)} \rceil}{\lceil \mathrm{SNR}^{M^2K^2+MK(K-2)M} \rceil}$ if $2M-1 \geq MK$ and $\rho = \frac{\lceil \mathrm{SNR}^{M^2K(K-1)} \rceil}{\lceil \mathrm{SNR}^{MK(MK-1)} \rceil}$ if $2M-1 \leq MK$ for sufficiently small $\epsilon(\approx 0)$. As shown in Fig. 3, the proposed distributed scheduling has a much better scaling law with the increase of M, K, and SNR values.

Fig. 4 shows the worst case feedback overheads per user, ρ_1 and ρ_2, according to SNR and M when $K = 3$. As shown in Fig. 4, the worst case feedback overheads per user exponentially decrease with SNR and the exponential decreasing order is determined by M^2. This result implicates that the feedback burden of each user is quite low since only a small number of receivers among the entire transmitter-receiver pairs in the network send the feedback signal in the proposed protocol.

Fig. 5(a) shows the link establishment probability of the proposed protocol for varying N when ϵ_k is set to be c for all k so that $\epsilon \geq \sum_{k=1}^{K-1} \epsilon_k = (K-1)c$. This figure shows that the link establishment probability becomes larger as N increases. On the contrary, as M and K increase, the link establishment probability becomes smaller due to the increase of interference. Note that if c is set to be larger, the link establishment can be higher for given N. However, sum-rate might decrease as c becomes larger because the leakage interference becomes larger. Therefore, there exists a trade-off between the link establishment probability and the achievable sum-rate for a given value of c. Fig. 5(b) shows the average number of feedback for varying N when the entire K user pairs are selected. As N increases, the average feedback number obviously increases, but the slope of the increase becomes gentler. While the average number of feedback becomes smaller as M increases, it becomes larger as K and c increases. If M increases, the received interference also increases and hence the number of receivers satisfying the threshold condition decreases. If c is high, the link is established with high probability but the average number of feedback also increases and the sum rate becomes smaller due to increased leakage interference. Note that ϵ is determined by required service quality in the network and remains unchanged over the selection process. For given ϵ, we can arbitrarily choose ϵ_k in the k-th selection step under the constraint $\epsilon \geq \sum_{k=1}^{K-1} \epsilon_k$. In this way, the total interference power received at each user after the k-th selection step is still maintained below ϵ. However, as shown in Fig 5(a) and 5(b), the choice of $\{\epsilon_k\}$ affects the link establishment probability and the average number of feedback for a finite number of N. This observation motivates a proper selection of $\{\epsilon_k\}$ by taking into account the priority among the average number of feedback and the link establishment probability, etc. Fig. 5(a) and Fig. 5(b) show that the proposed protocol can work even for a finite number of N although the achievable DoF is not equal to K.

We have investigated the effects of channel distribution by changing the variance of Rayleigh distribution. Fig. 6(a), 6(b), 7(a), and 7(b) show the link establishment probability, average number of feedback, leakage interference, and sum rate for different channel variances of the Rayleigh fading channel, respectively. The network area is given by a square $[0, 50] \times [0, 50]$, ϵ_1 = 6, ϵ_2 = 12, and $N \geq 10$. As the standard deviation σ of the Rayleigh fading channel increases, both the link establishment probability and the sum rate decreases, but the average number of feedback increases. This is because as shown in Fig. 7(a), interference power increases as the channel power gain becomes larger. However, interestingly,

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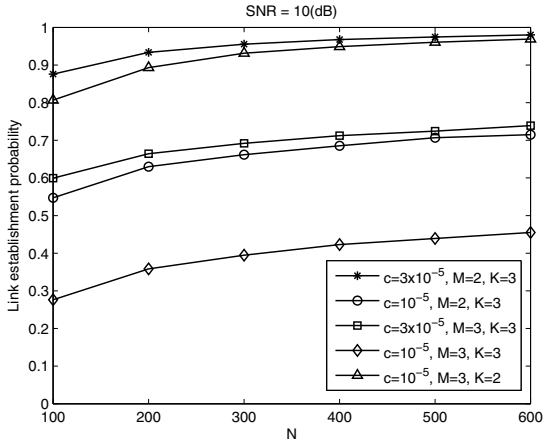
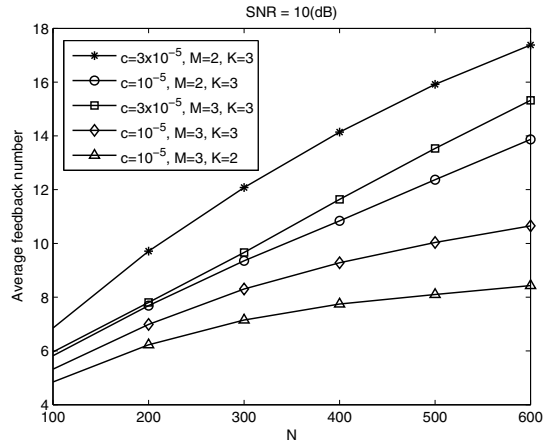
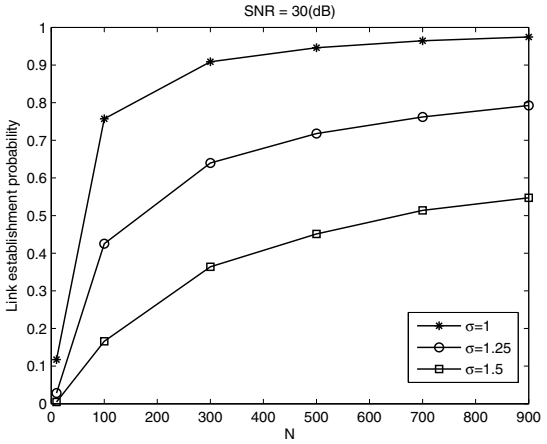


(a) Link establishment probability

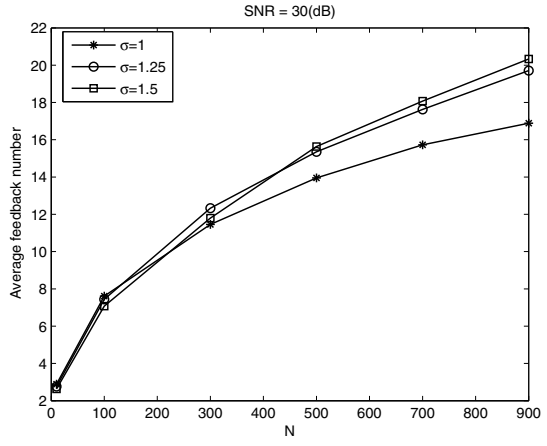


(b) Average number of feedback

Fig. 5. Link establishment probability and average number of feedback for varying N when the pathloss exponent is 4 and the network area is given by a square $[0, 100] \times [0, 100]$.



(a) Link establishment probability



(b) Average number of feedbacks

Fig. 6. The link establishment probability and average number of feedbacks for varying standard deviation of Rayleigh fading channel

the increasing speed of sum rate versus N becomes faster if σ is large because the total interference power is limited by the threshold levels ϵ_1 and ϵ_2 through the selection process even though interference power increases with σ, while the desired channel power gain increases with σ.

VII. Conclusions

In this paper, we proposed an opportunistic, sequential and distributed user scheduling protocol which achieves the maximum number of DoF MK in the (N, K)-user interference channel where each user is equipped with M antennas. In the proposed scheduling protocol, user pairs are selected if their both caused and received interference to/from the already selected user pairs are smaller than a certain threshold level. It was proven that when the codeword length is long enough, the proposed scheduling protocol achieves the maximum number of DoF MK with less stringent user scaling law $N = \omega\big(\mathrm{SNR}^{M^2K(K-1)}\big)$, compared to the centralized scheduling protocol which is known as the best scheme in the (N, K)-user interference channel. We also investigated feedback overheads of the proposed protocol for validating its practical feasibility. The numerical results showed that the required feedback overheads per user in the proposed protocol become negligible when SNR increases. It was also shown that our proposed scheme controls the interference much effectively than the conventional centralized protocol.

Appendix A
Proof of Lemma 2

In the $(k+1)$-th user selection step, the probability that both transmitter and receiver of one particular user pair $s \in S_k^c$ among the remaining N–k candidate transmitter-receiver pairs *simultaneously* satisfy the threshold condition ϵ_k (we call this "the link is established") can be written as

$$\Pr\left\{\sum_{n=1}^{M}\sum_{m=1}^{M}\sum_{j=1}^{k}|\mathbf{g}_j^{(n)H}\mathbf{h}_{s,j}^{(m)}|^2 \le \epsilon_k \mathrm{SNR}^{-1}\right\} \times \Pr\left\{\sum_{n=1}^{M}\sum_{m=1}^{M}\sum_{i=1}^{k}|\mathbf{g}_s^{(n)H}\mathbf{h}_{i,s}^{(m)}|^2 \le \epsilon_k \mathrm{SNR}^{-1}\right\} \quad (25)$$

$$= F_{J_k^s}\left(\epsilon_k \mathrm{SNR}^{-1}\right) F_{J_k^s}\left(\epsilon_k \mathrm{SNR}^{-1}\right) \quad (26)$$

$$= \left\{F_{J_k^s}(\epsilon_k \mathrm{SNR}^{-1})\right\}^2, \quad (27)$$

where $F_{J_k^s}(\cdot)$ is CDF of J_k^s following Chi-square distribution with $2kM^2$ degrees of freedom. Note that when the user pair s

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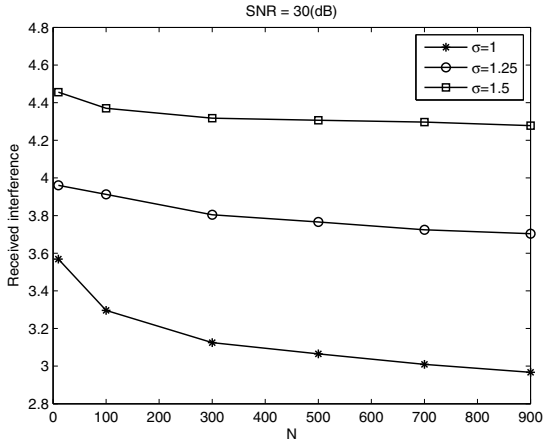
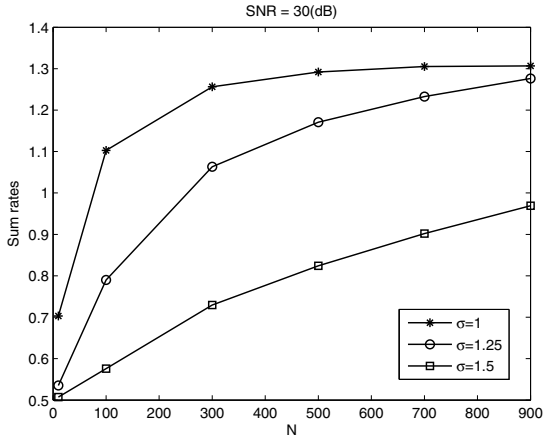


(a) Received interference



(b) Sum rates

Fig. 7. The received interference and sum rates for varying standard deviation of Rayleigh fading channel

satisfies the threshold condition ϵ_k, the aggregated interference from transmitter s to the receivers of already selected user pairs is lower than ϵ_k and the aggregated interference from transmitters of already selected user pairs to receiver s is lower than ϵ_k. Therefore, the probability that there is at least one transmitter-receiver pair simultaneously satisfying the threshold condition among $N-k$ candidate transmitter-receiver pairs becomes

$$\begin{aligned}
&\Pr\{\text{at least one link is established among } N-k \text{ transmitter-receiver pairs}\} && (28)\\
&= 1-\Pr\{\text{link is not established}\} && (29)\\
&= 1-\binom{N-k}{0}\left\{\left(F_{J_k^s}\left(\epsilon_k \mathrm{SNR}^{-1}\right)\right)^2\right\}^0 \times \left\{1-\left(F_{J_k^s}\left(\epsilon_k \mathrm{SNR}^{-1}\right)\right)^2\right\}^{N-k} && (30)\\
&= 1-\left\{1-\left(F_{J_k^s}\left(\epsilon_k \mathrm{SNR}^{-1}\right)\right)^2\right\}^{N-k}. && (31)
\end{aligned}$$

APPENDIX B
PROOF OF THEOREM 1

From (5), our proposed protocol achieves the maximum number of DoF MK, if the interference

$$\sum_{i=1, i\neq j}^{K}\sum_{m=1}^{M}\gamma_{i,j}|\mathbf{g}_j^{(n)H}\mathbf{h}_{i,j}^{(m)}|^2\mathrm{SNR} \quad (32)$$

has a finite value $\epsilon > 0$ which is independent of SNR for given all $1 \le j \le K$ and $1 \le n \le M$. The number of the achievable DoF can be written as

$$\sum_{j=1}^{N} d_j = \mathrm{P}_{\text{prop}} \cdot MK, \quad (33)$$

where

$$\mathrm{P}_{\text{prop}} = \lim_{\mathrm{SNR}\to\infty} \Pr\Bigg\{ \sum_{i=1, i\neq j}^{K}\sum_{m=1}^{M}\gamma_{i,j}|\mathbf{g}_j^{(n)H}\mathbf{h}_{i,j}^{(m)}|^2\mathrm{SNR} \le \epsilon \text{ for all } j \in \{1,\cdots,K\},\ n \in \{1,\cdots,M\} \Bigg\}, \quad (34)$$

where $\Pr(\cdot)$ denotes the probability. The probability P_{prop} in (34) is lower bounded as in the following at the top of next page, where $\gamma_{\max} = \max_{i,j\in\{1,\cdots,K\}, i\neq j}\gamma_{i,j}$ and $\epsilon = 2(\epsilon_1 + \epsilon_2 + \cdots + \epsilon_{K-1})$.

The inequality (35) holds because the constraint (34) that each antenna at the selected users has the interference lower than ϵ is relaxed into the constraint that the sum of interference of all selected users is lower than ϵ (i.e., $\Pr\{A \le \epsilon\}\Pr\{B \le \epsilon\} \ge \Pr\{A+B \le \epsilon\}$). The inequality (37) comes from the fact that each pathloss term is strictly less than 1. Note that each probability term in the last inequality corresponds to each step of user pair selection in our proposed scheduling protocol.

Each probability term in (40) is equivalent to the probability that at least one transmitter-receiver pair satisfies the threshold condition in each user selection step (i.e., link establishment probability). Using Lemma 2, the probability in (40) is rewritten by

$$\lim_{\mathrm{SNR}\to\infty}\prod_{k=1}^{K-1}\left[1-\left\{1-\left(F_{J_k^s}\left(\epsilon_k \mathrm{SNR}^{-1}\right)\right)^2\right\}^{N-k}\right] \quad (41)$$

$$\overset{(a)}{\ge} \lim_{\mathrm{SNR}\to\infty}\prod_{k=1}^{K-1}\left[1-\left\{1-\left(\bar{C}_k^1\right)^2 \mathrm{SNR}^{-2kM^2}\right\}^{N-k}\right] \quad (42)$$

where $\bar{C}_k^1 = C_k^1 {\epsilon_k}^{kM^2}$ and $C_k^1 = \frac{2^{-kM^2}}{kM^2\cdot\Gamma(kM^2)}e^{-\epsilon_k \mathrm{SNR}^{-1}/2}$ for $1 \le k \le K-1$. Since the J_k^s has Chi-square distribution with $2kM^2$ degrees of freedom, the inequality (a) is easily derived by using the lowerbound in Lemma 1. Each multiplication term in (42), which corresponds to the $(k+1)$-th user pair selection step of our proposed protocol, is generalized by $1-\left\{1-\left(\bar{C}_k^1\right)^2 \mathrm{SNR}^{-2kM^2}\right\}^{N-k}$ for $1 \le k \le K-1$. Since $\bar{C}_k^1$ is considered as a constant when SNR goes to infinity, $1-\left\{1-\left(\bar{C}_k^1\right)^2 \mathrm{SNR}^{-2kM^2}\right\}^{N-k}$ goes to 0 as SNR goes to infinity for a *finite* value of N. Consequently, the right hand side of the inequality (a) goes to zero for a *finite* value of N.

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$$\mathrm{P}_{\mathrm{prop}} \geq \lim_{\mathrm{SNR}\to\infty} \Pr\left\{ \sum_{j=1}^{K}\sum_{n=1}^{M}\sum_{i=1,i\neq j}^{K}\sum_{m=1}^{M} \gamma_{i,j}|\mathbf{g}_j^{(n)H}\mathbf{h}_{i,j}^{(m)}|^2\mathrm{SNR} \leq \epsilon \right\} \tag{35}$$

$$\geq \lim_{\mathrm{SNR}\to\infty} \Pr\left\{ \sum_{j=1}^{K}\sum_{n=1}^{M}\sum_{i=1,i\neq j}^{K}\sum_{m=1}^{M} \gamma_{\max}|\mathbf{g}_j^{(n)H}\mathbf{h}_{i,j}^{(m)}|^2\mathrm{SNR} \leq \epsilon \right\} \tag{36}$$

$$\geq \lim_{\mathrm{SNR}\to\infty} \Pr\left\{ \sum_{j=1}^{K}\sum_{n=1}^{M}\sum_{i=1,i\neq j}^{K}\sum_{m=1}^{M} |\mathbf{g}_j^{(n)H}\mathbf{h}_{i,j}^{(m)}|^2 \leq \epsilon\mathrm{SNR}^{-1} \right\} \tag{37}$$

$$= \lim_{\mathrm{SNR}\to\infty} \Pr\left\{ \sum_{n=1}^{M}\sum_{m=1}^{M} \left(|\mathbf{g}_1^{(n)H}\mathbf{h}_{2,1}^{(m)}|^2 + |\mathbf{g}_2^{(n)H}\mathbf{h}_{1,2}^{(m)}|^2 \right) + \sum_{n=1}^{M}\sum_{m=1}^{M} \left(\sum_{j=1}^{2} |\mathbf{g}_j^{(n)H}\mathbf{h}_{3,j}^{(m)}|^2 + \sum_{i=1}^{2} |\mathbf{g}_3^{(n)H}\mathbf{h}_{i,3}^{(m)}|^2 \right) + \cdots \right.$$
$$\left. + \sum_{n=1}^{M}\sum_{m=1}^{M} \left(\sum_{j=1}^{K-1} |\mathbf{g}_j^{(n)H}\mathbf{h}_{K,j}^{(m)}|^2 + \sum_{i=1}^{K-1} |\mathbf{g}_K^{(n)H}\mathbf{h}_{i,K}^{(m)}|^2 \right) \leq 2\left(\epsilon_1 + \epsilon_2 + \cdots + \epsilon_{K-1}\right)\mathrm{SNR}^{-1} \right\} \tag{38}$$

$$\geq \lim_{\mathrm{SNR}\to\infty} \prod_{k=1}^{K-1} \Pr\left\{ \sum_{n=1}^{M}\sum_{m=1}^{M} \left(\sum_{j=1}^{k} |\mathbf{g}_j^{(n)H}\mathbf{h}_{k+1,j}^{(m)}|^2 + \sum_{i=1}^{k} |\mathbf{g}_{k+1}^{(n)H}\mathbf{h}_{i,k+1}^{(m)}|^2 \right) \leq 2\epsilon_k\mathrm{SNR}^{-1} \right\} \tag{39}$$

$$\geq \lim_{\mathrm{SNR}\to\infty} \prod_{k=1}^{K-1} \Pr\left\{ \sum_{n=1}^{M}\sum_{m=1}^{M}\sum_{j=1}^{k} |\mathbf{g}_j^{(n)H}\mathbf{h}_{k+1,j}^{(m)}|^2 \leq \epsilon_k\mathrm{SNR}^{-1}, \sum_{n=1}^{M}\sum_{m=1}^{M}\sum_{i=1}^{k} |\mathbf{g}_{k+1}^{(n)H}\mathbf{h}_{i,k+1}^{(m)}|^2 \leq \epsilon_k\mathrm{SNR}^{-1} \right\}, \tag{40}$$

We now show that if the total number of transmitter-receiver pairs (i.e., network size) N scales at least as $\omega\left(\mathrm{SNR}^{2kM^2}\right)$ for each k, $\mathrm{P}_{\mathrm{prop}}$ goes to 1. Making each probability in (42) be one implicates that at least one transmitter-receiver pair surely satisfies the threshold conditions at both the transmitter and the receiver in each user pair selection step and one transmitter-receiver pair is surely selected in each selection step by our proposed scheduling protocol. For any constant $a > 0$ and $b > 0$, it is easily derived by using the relation of $\lim_{x\to\infty}\left(1-\frac{c}{x}\right)^x = \frac{1}{e^c}$ that if the total number of transmitter-receiver pairs N grows as fast as $\omega\left(\mathrm{SNR}^{2ka}\right)$, then $\lim_{\mathrm{SNR}\to\infty}\left\{1 - b\ \mathrm{SNR}^{-2ka}\right\}^{N-k}$ goes to 0.

Consequently, each probability term represented by $1 - \left\{1 - \left(\bar{C}_k^1\right)^2 \mathrm{SNR}^{-2kM^2}\right\}^{N-k}$ in (42) can be 1 if the required transmitter-receiver pairs scale at least as $\omega\left(\mathrm{SNR}^{2kM^2}\right)$ for $1 \leq k \leq K-1$. Since the channel coefficients are mutually independent, the scaling law of the required number of transmitter-receiver pairs to achieve the maximum number of DoF MK is obtained by $N = \omega\left(\mathrm{SNR}^{2M^2+4M^2+\cdots+2(K-1)M^2}\right) = \omega\left(\mathrm{SNR}^{M^2K(K-1)}\right)$. Note that we found the required user scaling law by making at least one user pair is selected at each user selection step with probability 1. Therefore, as long as $N = \omega\left(\mathrm{SNR}^{M^2K(K-1)}\right)$, the probability that link is established at each user selection step is 1 and the probability that the protocol is reset is 0.

From the achievability proof, we conclude that if total communication pairs N scales at least as fast as $\omega(\mathrm{SNR}^{M^2K(K-1)})$, then the maximum number of DoF MK is always achievable in our proposed scheduling protocol with ZF receiver. Note that this resulting scaling law $\omega\left(\mathrm{SNR}^{M^2K(K-1)}\right)$ is much smaller than that in [25] for the maximum number of Dof MK when the number of antennas is $M > 1$ and the active number of users is $K > 1$.

REFERENCES

[1] A. B. Carlerial, "A case where interference does not reduce capacity," *IEEE Trans. Inf. Theory*, vol. 21, no. 5, pp. 569–570, Sep. 1975.

[2] H. Sato, "The capacity of the Gaussian interference channel under strong interference," *IEEE Trans. Inf. Theory*, vol. 27, no. 6, pp. 786–788, Nov. 1981.

[3] T. S. Han and K. Kobayashi, "A new achievable rate region for the interference channel," *IEEE Trans. Inf. Theory*, vol. 27, no. 1, pp. 49–60, Jan. 1981.

[4] R. Etkin, D. N. C. Tse, and H. Wang, "Gaussian interference channel capacity to within one bit," *IEEE Trans. Inf. Theory*, vol. 54, no. 12, pp. 5534–5562, Dec. 2008.

[5] S. Jafar and M. J. Fakherenddin, "Degrees of freedom for the MIMO interference channel," *IEEE Trans. Inf. Theory*, vol. 53, no. 7, pp. 2637–2642, July 2007.

[6] V. Cadambe and S. Jafar, "Interference alignment and degrees of freedom of the K-user interference channel," *IEEE Trans. Inf. Theory*, vol. 54, no. 8, pp. 3425–3441, Aug. 2008.

[7] J. H. Lee and W. Choi, "Achievable DoF and its user scaling law for opportunistic interference alignment in three-user MIMO interference channels," to appear in *IEEE Trans. Wireless Commun.*. Available: http://arxiv.org/abs/1109.6541.

[8] R. Knopp and P. Humlet, "Information capacity and power control in single cell multiuser communications," in *Proc. 1995 IEEE ICC*, pp. 331–335.

[9] P. Viswanath, D. N. C. Tse, and R. Laroia, "Opportunistic beamforming using dumb antennas," *IEEE Trans. Inf. Theory*, vol. 48, no. 6, pp. 1277–1294, June 2002.

[10] X. Li, E. K. P. Chong, and N. B. Shroff, "A framework for opportunistic scheduling in wireless networks," *Computer Netowkrs*, vol. 41, no. 3, pp. 451–474, Mar. 2003.

[11] X. Qin and R. Berry, "Exploiting multi-user diverrsity for medium access control in wireless networks," in *Proc. 2003 IEEE INFOCOM*, pp. 1084–1094.

[12] S. Borst, "User-level performance of channel-aware scheduling algorithms in wireless data networks," *IEEE/ACM Trans. Netw.*, vol. 13, no. 3, pp. 636–647, June 2005.

[13] M. J, Neely, "Super-fast delay tradeoffs for utility optimal fair scheduling in wireless networks," *IEEE J. Sel. Areas Commun.*, vol. 24, no. 8, pp. 1489–1501, Aug. 2006.

[14] W. Choi and J. G. Andrews, "The capacity gain from intercell scheduling in multi-antenna systems," *IEEE Trans. Wireless Commun.*, vol. 7, no. 2, pp. 714–725, Feb. 2008.

[15] Y. Huang and B. D. Rao, "An analytical framework for heterogeneous partial feedback design in heterogeneous multicell OFDMA networks," *IEEE Trans. Signal Process.*, vol. 61, no. 3, pp. 753–769, Feb. 2013.

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[16] D. Zheng, M. O.Pun, W. Ge, J. Zhang, and H. V. Poor, "Distributed opportunistic scheduling for ad hoc communications with imperfect channel information," *IEEE Trans. Wireless Commun.*, vol. 7, no. 12, pp. 5450–5460, Dec. 2008.
[17] D. Zheng, M. O.Pun, W. Ge, J. Zhang, and H. V. Poor, "Distributed opportunistic scheduling for ad-hoc network with random access:an optimal stopping approach," *IEEE Trans. Inf. Theory*, vol. 55, no. 1, pp. 205–222, Jan. 2009.
[18] Z. Wang, M. Ji, H. R. Sadjadpour, and J. J. Garcia-Luna-Aceves, "Opportunistic interference management increases the capacity of ad hoc networks," in *Proc. 2010 IEEE SECON*, pp. 1–9.
[19] S. Weber, J. G. Andrews and N. Jindal, "The effect of fading, channel inversion, and threshold scheduling in ad hoc networks," *IEEE Trans. Inf. Theory*, vol. 53, no. 11, pp. 4127–4149, Nov. 2007.
[20] C. H. Liu and J. G. Andrews, "Didstributed SIR-Aware Scheduling in Large-Scale Wireless Networks." Available: http://arxiv.org/abs/1107.1731.
[21] F. Bacelli, B. Blaszczyszyn and P. Mühlethaler, "Stochastic analysis of spatial and opportunistic Aloha," *IEEE J. Sel. Areas Commun.*, vol. 27, no. 7, pp. 1105–1119, Sep. 2009.
[22] D. Gesbert, S. G. Kiani, A. Gjendemsjø, and G. E. Øien, "Adaptation, coordination and distributed resource allocation in interference-limited wireless networks," *Proc. IEEE*, pp. 2393–2409, Dec. 2007.
[23] S. G. Kiani, D. Gesbert, A. Gjendemsjø, and G. E. Øien, "Distributed power allocation for interfering wireless links based on channel information partitioning," *IEEE Trans. Wireless Commun.*, vol. 8, no. 6, pp. 3004–3015, June 2009.
[24] A. Tajer and X. Wang, "(n, K)-user interference channels: degrees of freedom," in *Proc. 2011 IEEE ISIT*, pp. 1372–1376.
[25] A. Tajer and X. Wang, "(n, K)-user interference channels: degrees of Freedom," *IEEE Trans. Inf. Theory*, vol. 58, no. 8, pp. 5338–5353, Aug. 2012.
[26] D. E. Knuth, "Big omicron and big omega and big theta," *ACM SIGACT News*, pp. 18–24, April-June, 1967.
[27] B. C. Jung, D. H. Park, and W. Y. Shin, "Opportunistic interference mitigation achieves optimal degrees-of-freedom in wireless multi-cell uplink networks," *IEEE Trans. Commun.*, vol. 60, no. 7, pp. 1935–1944, July 2012.
[28] R. Tannious and A. Nosratinia, "Spectrally-efficient relay selection with limited feedback," *IEEE J. Sel. Areas Commun.*, vol. 26, no. 8, pp. 1419–1428, Oct. 2008.
[29] A. Bletsas, A. Khisti, D. P. Reed, and A. Lippman, "A simple cooperative diversity method based on network path selection," *IEEE J. Sel. Areas Commun.*, vol. 24, no. 3, pp. 659–672, Mar. 2006.
[30] A. Tajer and A. Nosratinia, "Opportunistic cooperation via relay selection with minimal information exchange," in *Proc. 2007 IEEE ISIT*, pp. 1926–1930.
[31] K. Azarian, H. E. Gamal, and P. Schniter, "On the achievable diversity-multiplexing tradeoff in half-duplex cooperative channels," *IEEE Trans. Inf. Theory*, vol. 51, no. 12, pp. 4152–4172, Dec. 2005.
[32] A. Özgür, O. Lévêque, and D. Tse, "Hierarchical cooperation achieves optimal capacity scaling in ad hoc networks," *IEEE Trans. Inf. Theory*, vol. 53, no. 10, pp. 3549–3572, Oct. 2007.
[33] O. Simeone, U. Spagnolini, Y. Bar-Ness, and S. H. Strogatz, "Distributed synchronization in wireless networks," *IEEE Signal Process. Mag.*, vol. 25, no. 5, pp. 81–97, Sep. 2008.
[34] Y.-C. Wu, Q. Chaudhari, and E. Serpedin, "Clock synchronization of wireless sensor networks," *IEEE Signal Process. Mag.*, vol. 28, no. 1, pp. 124–138, Jan. 2011.

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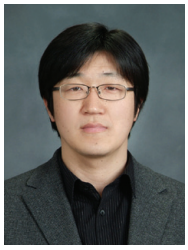

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