

# Random FH-OFDMA System Based on Statistical Multiplexing

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**Abstract**—We propose a random frequency hopping orthogonal frequency division multiple access (RFH-OFDMA) system based on statistical multiplexing for improving downlink user capacity. User capacity is defined as the maximum number of users in case that served with a basic data-rate in a cell. We compare the downlink user capacity of the proposed RFH-OFDMA system with that of the conventional frequency hopping OFDMA (FH-OFDMA) systems. User capacity is limited by either the number of subcarriers or other-cell interference (OCI). Simulation results show that the proposed RFH-OFDMA system can accommodate 262 users in a 3-sector based cell, while the conventional FH-OFDMA systems accommodate 51 users when the user channel activity is 0.1.

## I. INTRODUCTION

Recently, data traffic has rapidly increased in wireless communication systems. From this trend, data traffic will be expected to be dominant in future wireless systems. Data traffic is inherently bursty and generally exhibits a low user activity. Furthermore, there is more downlink traffic than uplink traffic. Several efficient downlink systems have been proposed to provide this data traffic in wireless link [1], [2].

Orthogonal frequency division multiplexing (OFDM) is one of the most promising techniques for high-speed data transmission over frequency selective fading channels and a number of multiple access schemes with OFDM techniques have been proposed including OFDM-TDMA, OFDM-FDMA, OFDM-CDMA, and FH-OFDMA [3]. Especially, Frequency hopping (FH)-OFDMA is useful for cellular networks because it does not cause intra-cell interference (ICI) and averages out other-cell interference (OCI). In FH-OFDMA, each user has an unique frequency hopping pattern (HP) to select a subcarrier or a group of subcarriers during each symbol time. Base station (BS) allocates each user an orthogonal HP in a call-setup process. Thus, multiple users do not occupy the same subcarrier at the same time.

A frequency domain scheduling (FDS) with adaptive modulation and coding (AMC) scheme using the channel information of each user and a dynamic channel allocation (DCA) scheme in TDMA have been considered for increasing the throughput in OFDM systems [4], [5]. However, these techniques require a large amount of feedback information and may not operate correctly in rapidly varying channel environments. Furthermore, these techniques based on scheduling is appropriate for high-rate data services among a small

number of users. If a large number of users demand low-rate data services with low channel activities, then a BS should transmit a lot of signaling information, which includes subcarrier allocation, modulation and coding format, pending data to specific users, in order to maintain the connections.

*Statistical multiplexing* is a method in which users occupy a given link only when they send packets. Statistical multiplexing schemes contrast with scheduling-based ones because they do not control user data transmissions. In this paper, we propose a random frequency hopping (RFH)-OFDMA system based on statistical multiplexing for increasing the downlink user capacity. *User capacity* is defined as the maximum number of users supported with a basic data-rate by a BS. Using the proposed scheme, more users than the number of subcarriers can be supported efficiently when user channel activity is relatively low.

The remainder of this paper is organized as follows: In Section II we analyze user capacity of the conventional FH-OFDMA systems considering the available number of subcarriers in a cell, OCI, and user activity. Moreover, we propose a random FH (RFH)-OFDMA system based on statistical multiplexing and analyze the user capacity of the proposed system in Section III. A numerical example for the performance of the proposed system is described in Section IV. Finally, conclusions are presented in Section V.

## II. USER CAPACITY OF FH-OFDMA SYSTEMS

Conventional approaches to calculating the user capacity of FDMA systems were based on the trade-off between frequency reuse factor and signal-to-interference ratio (SIR) [6]. In the conventional capacity analysis, all other-cell frequency channels are assumed to be used by users since the system utilizes a narrow band for users and the situation is probable even if the offered load is less than the system capacity. Thus, user capacity is equivalent to the available number of subcarriers which is determined by the frequency reuse factor. In addition, this approach did not consider the user channel activities because each user is assumed to be always active during a call, which is valid if the target service is voice with a relatively high channel activity of approximately 0.5. However, when we consider data traffic as a main service, the user channel activity may be below 0.2. In this case, users may not demand data in the most portion of time in downlink.

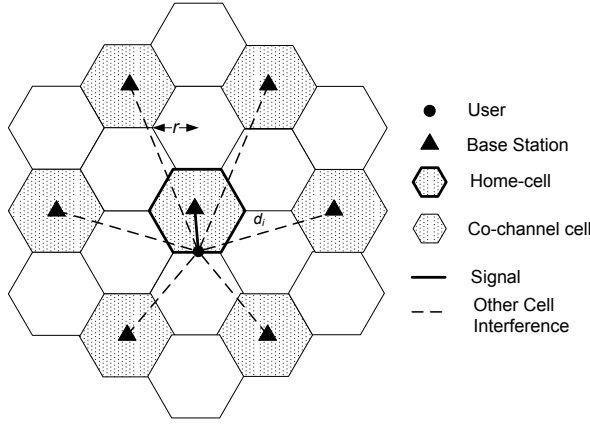


Fig. 1. System model

Fig. 1 shows a system model of FH-OFDMA systems. In this model, we assume that the frequency reuse factor is 3 and the cell radius is  $r$ . Of course, the frequency reuse factor can be 1 and sectorized antennas can be applied. Users are assumed to be located at the cell boundary and dark cells indicate the co-channel cells. The cells except for the co-channel cells do not cause interference to the home-cell because they use the different frequency bands from that of the home-cell. The solid line in Fig. 1 represents the signal transmitted by home-cell BS and the dotted lines represent OCI from the co-channel cell BSs.

First of all, the received signal-to-interference and noise ratio (SINR) should be analyzed to derive the downlink user capacity of FH-OFDMA systems considering the channel activity of the home-cell users and the offered load of other-cells. The received SINR in downlink can be expressed as [7]:

$$\frac{S}{I_0 + N_0} \geq \frac{P_T \cdot r^{-\alpha} \cdot X_0}{\left\{ \lambda \left( \frac{\bar{v}k + \rho M}{M} \right) \sum_{i=1}^{i_0} P_T \cdot d_i^{-\alpha} \cdot X_i \right\} + N_0}, \quad (1)$$

where

- $\alpha$  Path-loss exponent
- $d_i$  Distance between user and the  $i$ -th co-channel cell BS
- $I_0$  OCI at the receiver
- $i_0$  Number of co-channel cells
- $k$  Average number of users in each cell
- $\lambda$  Sector antenna factor (e.g.  $\lambda = 1$  for a omni-antenna case and  $\lambda = 1/3$  for 3-sector antenna case)
- $\rho$  Proportion of signaling overhead for synchronization and channel estimation.
- $M$  Available number of subcarriers per cell
- $N_0$  Thermal noise at the receiver
- $P_T$  Transmit power per subcarrier
- $r$  Cell radius
- $\bar{v}$  Average user channel activity
- $X_i$  Shadowing factor from the  $i$ -th co-channel cell BS
- $X_0$  Shadowing factor from home-cell BS

We assume that all users have the same user channel activity and the same number of users exists in each cell. Besides, all users in the home-cell are assumed to be at the cell boundary for the worst case. The signaling subchannel has a channel activity of 1 and  $M$  is related to cluster size. If we neglect thermal noise, the received  $E_b/I_0$  is expressed as

$$\frac{E_b}{I_0} = \frac{S/I_0}{R^{FEC} \cdot \mu}, \quad (2)$$

where  $R^{FEC}$  and  $\mu$  denote the channel code rate and the modulation order, respectively. If we assume that the shadowing factors have a median value of 1, Eqn. (2) can be approximated as

$$\frac{E_b}{I_0} \simeq \frac{\beta q^\alpha}{6\mu\lambda R^{FEC} \left( \frac{\bar{v}k + \rho M}{M} \right)}, \quad (3)$$

where

$$q = \frac{D}{r} \quad \text{and} \quad \beta = \frac{1}{1 + \left( \frac{1}{\sqrt{3}} \right)^\alpha + \left( \frac{1}{\sqrt{4}} \right)^\alpha + \left( \frac{1}{\sqrt{7}} \right)^\alpha + \dots}, \quad (4)$$

where  $D$  denotes the distance between the home-cell BS and the co-channel cell BS in the first tier. The lower bound of the user capacity is obtained when the OCI has the maximum value, that is, all user traffic subcarriers of the co-channel cells carry user data. The lower bound of the user capacity from OCI is approximately given as

$$\begin{aligned} C_{OCI, FH} &\simeq \left\{ \frac{\left( \frac{E_b}{I_0} \right)_{maxOCI}}{\bar{v} \left( \frac{E_b}{I_0} \right)_{required}} \right\} (1 - \rho)M \\ &= \frac{\beta q^\alpha (1 - \rho)M}{6\mu\lambda R^{FEC} \left\{ \bar{v} \left( \frac{E_b}{I_0} \right)_{required} \right\}}, \end{aligned} \quad (5)$$

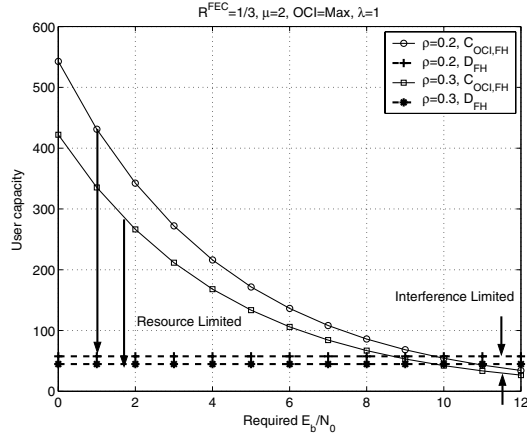
where  $(E_b/I_0)_{required}$  is the required  $E_b/I_0$  at the receiver for a specific data rate. In addition, the available number of subcarriers for user traffic in each cell is written as

$$D_{FH} = (1 - \rho)M. \quad (6)$$

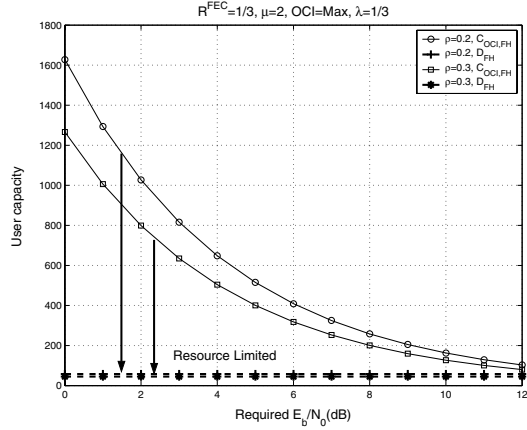
The user capacity of FH-OFDMA systems is expressed as

$$C_{FH} = \min\{C_{OCI, FH}, D_{FH}\}. \quad (7)$$

Fig. 2 shows the lower bound user capacities of the FH-OFDMA systems for varying the required  $E_b/N_0$ . The available number of subcarriers per cell ( $M$ ) is assumed to be 64 and each user utilizes one subcarrier to communicate with BS at a specific data-rate. Furthermore, the frequency reuse factor is assumed to be one. The result shows that user capacity is limited by OCI if the required  $E_b/N_0$  is high (*interference limited* situation), but it is limited by the number of subcarriers if the required  $E_b/N_0$  is low (*resource limited* situation) [8]. If the user capacity is limited by interference, the system can not accommodate new users even though the available number of subcarriers remains sufficient. On the contrary, the user capacity is limited by the available number of subcarriers, the



(a)  $\bar{\nu} = 0.2$ ,  $\lambda = 1$



(b)  $\bar{\nu} = 0.2$ ,  $\lambda = 1/3$

Fig. 2. User capacities of the FH-OFDMA Systems

system can not support more users than the available number of subcarriers even though OCI is small enough to accommodate more users. The resource limited situation occurs frequently as the user channel activity ( $\bar{\nu}$ ), OCI, and the required  $E_b/N_0$  decrease.

### III. RANDOM FREQUENCY HOPPING OFDMA USING STATISTICAL MULTIPLEXING

#### A. RFH-OFDMA

In conventional FH-OFDMA systems, BS does not allocate the same subcarrier to different users at the same time and there exist  $D_{FH}$  HPs, each of which is allocated to each user. The allocated HP corresponds to a physical channel in the conventional FH-OFDMA systems. If user capacity is limited by the resource, the number of physical channels should be increased to support more users. In particular, if the user channel activity is low, it causes a low channel utilization in the conventional FH-OFDMA systems. On the contrary, in order to increase the channel utilization, an RFH-OFDMA system using statistical multiplexing is proposed and it allows multiple users to select the same subcarrier at the same time. BS allocates a random HP to each user during the initial

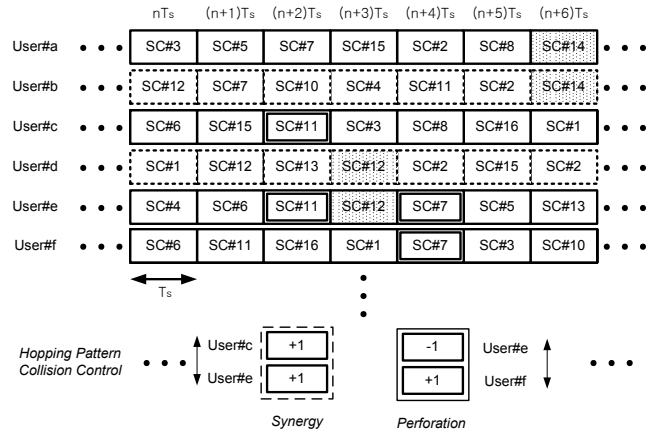


Fig. 3. Block diagram of the proposed RFH-OFDMA

association procedure and HP can be generated based on the user specific number, such as electronic serial number (ESN).

Fig. 3 shows the block diagram of the proposed RFH-OFDMA based on statistical multiplexing.  $T_s$  stands for the OFDM symbol time including the cyclic prefix and each user changes the subcarrier (SC) according to HP, which may make a HP collision. However, most of users may be inactive because of low channel activities. User #b and user #d are inactive in Fig. 3 but they follow their HPs during a session. In this case, HP collisions with inactive users do not affect the performance of the *active* user. The dark parts in Fig. 3 indicate this type of collision.

When an HP collision among the *active* users occurs, in the conventional FH-CDMA systems, it is considered as an inevitable interference (*hit*) in case that all users are asynchronous each other [9]. However, the HP collision can be detected by BS in a synchronous downlink environment. If it occurs, BS compares user data experiencing HP collisions and determines whether all user data with the same HP collisions are the same or not. If all the corresponding data are the same, the collision needs not be controlled and all the colliding symbols of different users are transmitted with one symbol energy, which results in an energy gain. On the contrary, if all data with the same HP collisions are not the same, all the corresponding data symbols are not transmitted (*perforated*) during the symbol time. Thus, in the proposed system, the collision, or hit, causes not ICI but information loss, and it can be recovered by proper channel coding techniques with additional energy.

The *hopping pattern collision* probability of the RFH-OFDMA system is written as:

$$P_c = 1 - \left\{ 1 - \frac{\bar{\nu}}{(1-\rho)M} \right\}^{K-1}, \quad (8)$$

where  $K$  is the number of users in a cell, which can be larger than  $M$ . For a given channel activity  $\bar{\nu}$ ,  $P_c$  increases as the number of users increases [10]. Eq. (8) includes only the collision among the active users.

### B. User capacity

As noted before, when a HP collision occurs in the proposed system, the colliding symbols can be controlled by using synergy and perforation. Synergy provides an energy gain to the receiver but the perforation yields information loss and the required  $E_b/N_0$  increases at the receiver. Therefore, the lower bound of the user capacity due to OCI in the proposed RFH-OFDMA system can be approximately expressed as:

$$C_{OCI,RFH} \simeq \frac{\beta q^\alpha (1 - \rho) M}{6\mu\lambda R^{FEC} \left[ \bar{v} \left\{ \left( \frac{E_b}{I_0} \right)_{required} + \Delta P \right\} \right]}, \quad (9)$$

where  $\Delta P$  is the additional energy which compensates for the HP collision probability  $P_c$ . The additional energy increases as the  $P_c$  increases and  $C_{OCI,RFH}$  decreases. Furthermore, the maximum available number of subcarriers in the proposed system for a given HP collision probability is written as:

$$D_{RFH} = 1 + \frac{\log(1 - P_c)}{\log \left\{ 1 - \frac{\bar{v}}{(1 - \rho)M} \right\}} \quad (10)$$

from Eqn. (8).

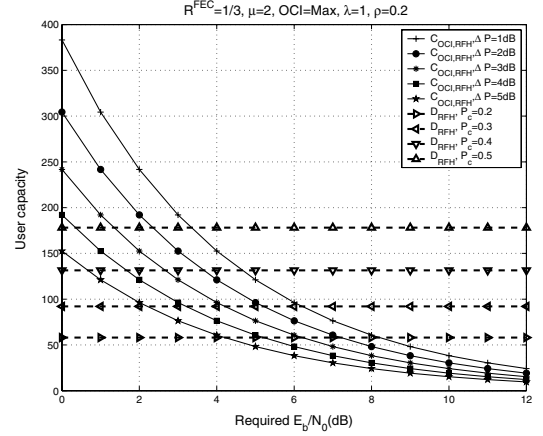
Note that  $D_{RFH}$  is not a constant and varies according to the user channel activity ( $\bar{v}$ ) and acceptable HP collision probability ( $P_c$ ). The acceptable HP collision probability is determined by the channel coding capability and the target frame error rate (FER). Thus, the user capacity of the proposed RFH-OFDMA system is expressed as:

$$C_{RFH} = \min\{C_{OCI,RFH}, D_{RFH}\}. \quad (11)$$

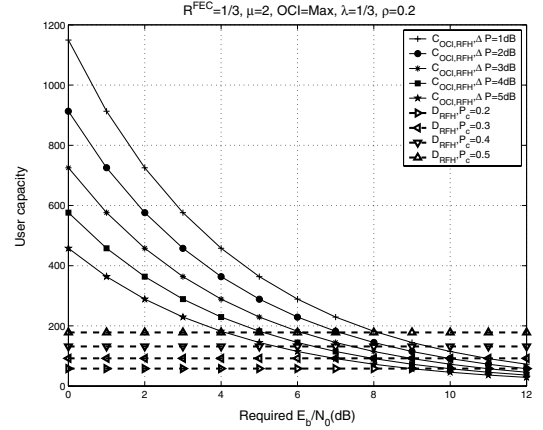
Fig. 4 shows the user capacities of the proposed RFH-OFDMA system for varying the additional energy and the acceptable HP collision probability. The available number of subcarriers per cell ( $M$ ) and the frequency reuse factor are assumed to be 64 and 1, respectively. The horizontal axis represents the required  $E_b/N_0$  when the HP collision probability is zero and the vertical axis indicates the downlink user capacities. In addition, the solid lines show  $C_{OCI,RFH}$  and the dotted lines represent  $D_{RFH}$ . The user capacity can be limited by either  $C_{OCI,RFH}$  or  $D_{RFH}$ . As the collision probability increases, the additional energy for a given FER requirement may also increase. If we can reduce  $\Delta P$  for a given  $P_c$ , the user capacity can be improved. Note that  $C_{OCI,RFH}$  decreases as the additional energy due to the HP collision effect increases but  $D_{RFH}$  increases as the acceptable HP collision probability increases. Therefore, there exists a trade-off between  $C_{OCI,RFH}$  and  $D_{RFH}$ . One critical design issue of the proposed RFH-OFDMA system is to reduce the  $(E_b/I_0)_{required}$  and  $\Delta P$ , which are highly related to the channel code performance. In Section IV, we show a practical system example.

### IV. NUMERICAL EXAMPLE

We consider a multipath fading channel with an exponential delay profile where each delay component is independently Rayleigh-distributed. The root-mean-square (RMS) value of delay spread is  $5\mu s$  and the maximum delay spread is  $25\mu s$ .



(a)  $\bar{v} = 0.2, \lambda = 1$



(b)  $\bar{v} = 0.2, \lambda = 1/3$

Fig. 4. User capacities of the proposed RFH-OFDMA systems.

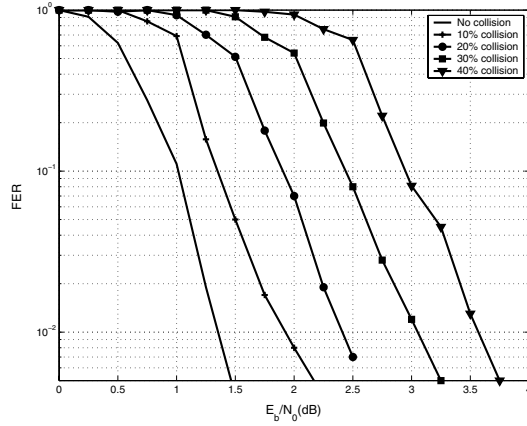
OFDM parameters are set as follows:

- Center frequency : 1.9 GHz
- Channel BW : 5 MHz
- Number of subcarriers : 1024
- Number of used subcarriers : 768
- Effective symbol duration : 204.8  $\mu s$
- Cyclic prefix : 25.6  $\mu s$

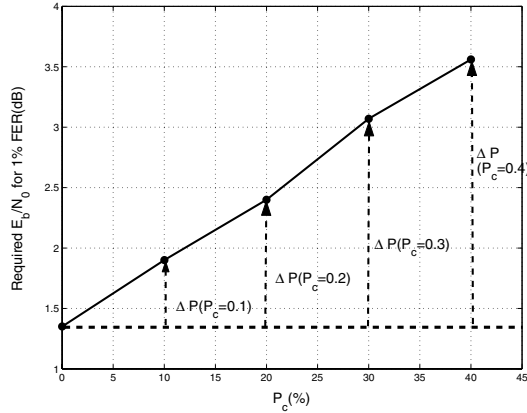
These parameters are based on a wide-area cellular-like system with target data rates of 10 to 20 Mbps [11].

We assume that each user uses 12 subcarriers at a time and all used subcarriers are divided into 12 groups (*subband*). Thus, each subband consists of 64 subcarriers. User data is transmitted according to the random HPs allocated in each subband. In this case, the available number of subcarriers ( $M$ ) is 64. In addition, QPSK is used for data modulation and turbo codes (a code rate of 1/3) are applied as a channel code. Therefore, each user's data rate is approximately 35 kbps. The turbo codes in 3GPP specifications are considered with a decoder using a maximum-a-posteriori (MAP) algorithm with an maximum iteration number of 8 [1] and the length of a frame is 680 bits.

Fig. 5(a) shows the FER performance of the proposed RFH-



(a) FER versus  $E_b/N_0$



(b) Required  $E_b/N_0$  for 1% FER and  $\Delta P$

Fig. 5. FER performance of the proposed RFH-OFDMA system.

OFDMA system. The required  $E_b/N_0$  for an FER of 0.01 is approximately 1.35 dB when there exists no HP collision. As the HP collision probability increases, the FER performance becomes worse. Fig. 5(b) shows the required  $E_b/N_0$  for 1% FER and the additional energy  $\Delta P$ . As noted before,  $\Delta P$  is mainly determined by the channel coding performance. If we use worse channel codes than the turbo codes or increase the code rate of channel codes (i.e., 1/2 and 2/3),  $\Delta P$  increases. When the HP collision probability is 0.4, the additional energy for 1% FER is 2.5 dB and  $(E_b/I_0)_{\text{required}}$  is 1.35 dB. Thus, the user capacity of the proposed RFH-OFDMA system is approximately 132 ( $\lambda = 1$  and  $\lambda = 1/3$ ) from Fig. 4. We assume that both  $\bar{v}$  and  $\rho$  are 0.2. The user capacity is much larger than that of FH-OFDMA system, which is approximately 50 in the same environment from Fig. 2. Table I summarizes the user capacity of the conventional FH-OFDMA system and that of the proposed RFH-OFDMA system for varying the user channel activity and the required  $E_b/N_0$ .

## V. CONCLUSIONS

In this paper, we propose an RFH-OFDMA system based on statistical multiplexing and analyze the user capacity of the conventional FH-OFDMA and the proposed RFH-OFDMA

TABLE I  
USER CAPACITIES OF THE CONVENTIONAL FH-OFDMA SYSTEM AND  
THE PROPOSED RFH-OFDMA SYSTEM.

$R^{FEC} = 1/3, \mu = 2, \rho = 0.2, \text{OCI} = Max$ $P_c = 0.4, \Delta P = 3dB, (E_b/I_0)_{\text{required}} = 1.35dB$				
	$\lambda = 1$		$\lambda = 1/3$	
$\bar{v}$	FH	RFH	FH	RFH
0.1	51.2	262.3	51.2	262.3
0.2	51.2	131.5	51.2	131.5
0.5	51.2	53.0	51.2	53.0
0.8	51.2	33.4	51.2	33.4

$R^{FEC} = 1/3, \mu = 2, \rho = 0.2, \text{OCI} = Max$ $P_c = 0.4, \Delta P = 3dB, (E_b/I_0)_{\text{required}} = 6.0dB$				
	$\lambda = 1$		$\lambda = 1/3$	
$\bar{v}$	FH	RFH	FH	RFH
0.1	51.2	121.5	51.2	262.3
0.2	51.2	60.7	51.2	131.5
0.5	48.5	24.3	51.2	53.0
0.8	30.3	15.2	51.2	33.4

systems. The user capacity of the FH-OFDMA system is limited by OCI or the number of available resources of the system. The proposed RFH-OFDMA system utilizes the statistical multiplexing allowing users to select the same subcarrier at the same time and it can accommodate more users than the available number of resources. When the user channel activity is low, the proposed RFH-OFDMA system yields a higher user capacity than that of the conventional FH-OFDMA systems.

## ACKNOWLEDGMENTS

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